

ZTF-FCT
Zientzia eta Teknologia Fakultatea
Facultad de Ciencia y Tecnología



Universidad del País Vasco
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Symmetry of commensurate magnetic structures: Magnetic space groups

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WHAT IS SYMMETRY?

A symmetry operation in a solid **IS NOT** *only* a more or less complex transformation leaving the system invariant.... **But it MUST fulfill that the resulting constraints can only be broken through a phase transition.**

A well defined symmetry operation (in a thermodynamic system) must be maintained when scalar fields (temperature, pressure,...) are changed, except if a phase transition takes place.

“symmetry-forced” means : “forced for a thermodynamic phase
“symmetry-allowed” means : “allowed within a thermodynamic phase”

Symmetry-dictated properties can be considered symmetry “protected”

Reminder of symmetry in non-magnetic structures

Space Group:
Pnma

Lattice parameters:
5.7461 7.6637 5.5333 90.000 90.000 90.000

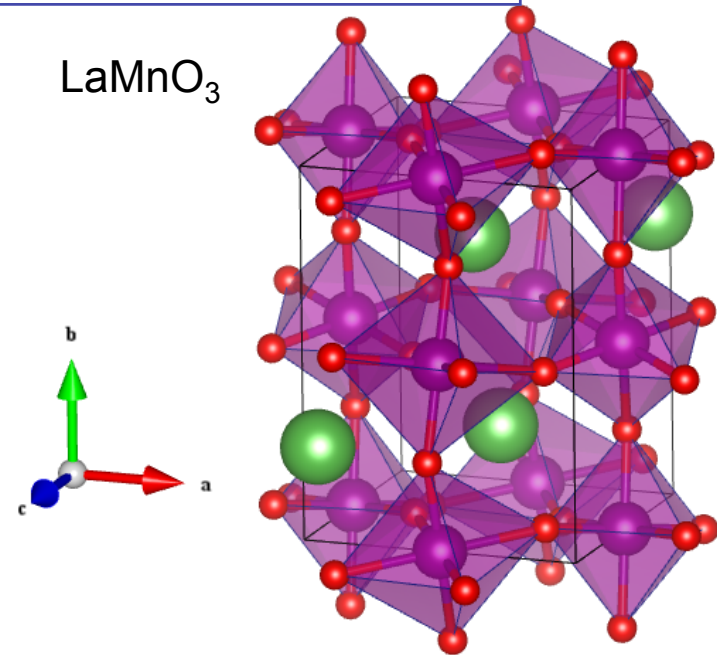
Atomic positions of asymmetric unit:

La1 0.05130 0.25000 -0.00950

Mn1 0.00000 0.00000 0.50000

O1 0.48490 0.25000 0.07770

O2 0.30850 0.04080 0.72270



Reminder of symmetry in non-magnetic structures

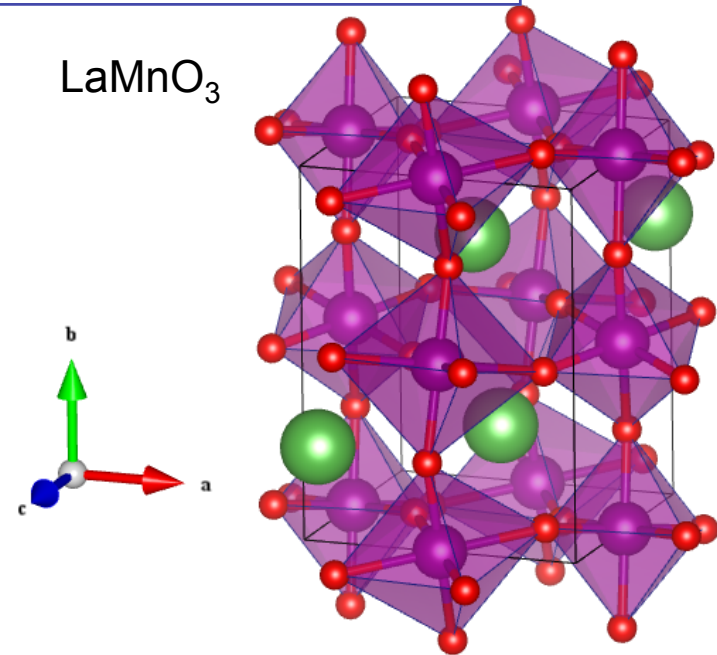
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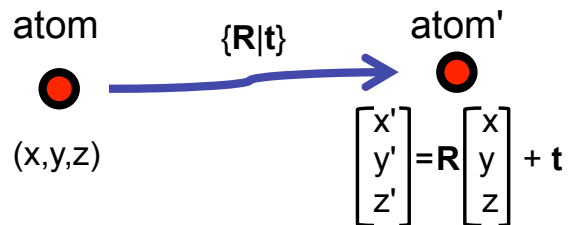
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LaMnO₃



Space Group: set of operations {R|t}

for all atoms:



{R|t}: R - rotation or rotation+plus inversion
t - translation

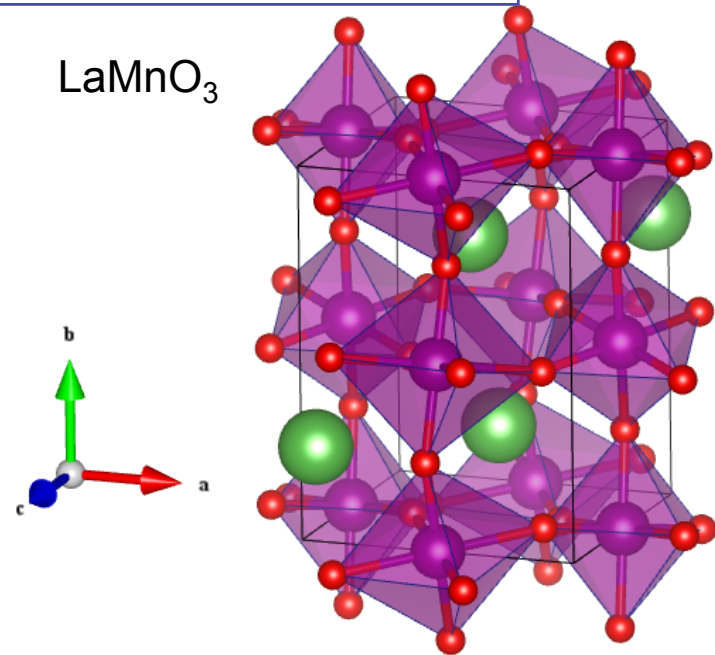
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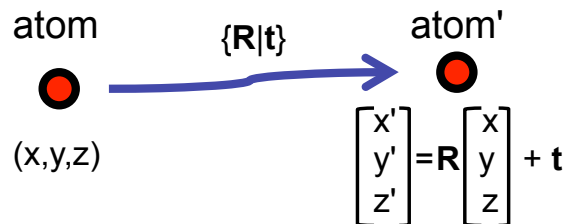
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Pnma: 8 related positions for a general position:

(x,y,z)	$(-x+1/2,-y,z+1/2)$	$(-x,y+1/2,-z)$	$(x+1/2,-y+1/2,-z+1/2) == \{2x \frac{1}{2} \frac{1}{2} \frac{1}{2}\}$
$(-x,-y,-z)$	$(x+1/2,y,-z+1/2)$	$(x,-y+1/2,z)$	$(-x+1/2,y+1/2,z+1/2) == \{m_x \frac{1}{2} \frac{1}{2} \frac{1}{2}\}$

4 related positions for a special position of type $(x, \frac{1}{4}, z)$: **special positions are tabulated:**

$(x, 1/4, z)$ $(-x+1/2, 3/4, z+1/2)$ $(-x, 3/4, -z)$ $(x+1/2, 1/4, -z+1/2)$ **Wyckoff positions or orbits**

Seitz Notation

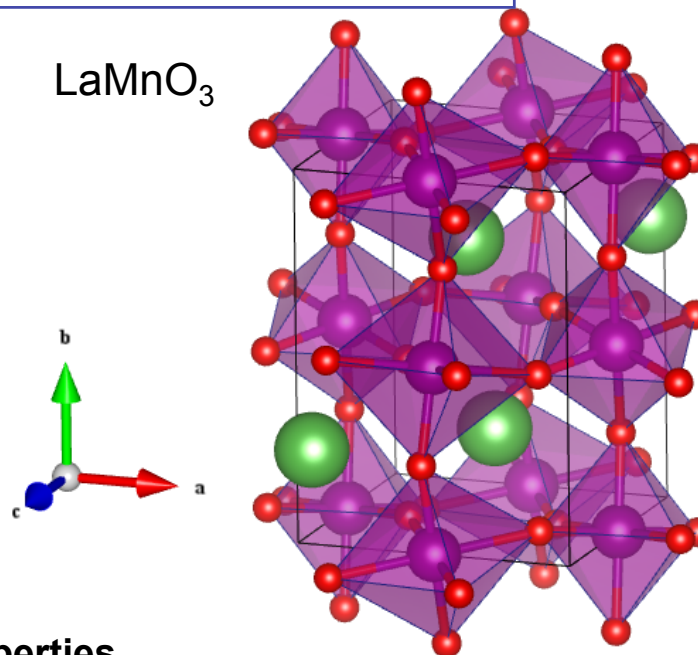
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Relations among atoms from the space group:
more than "geometrical", they are "thermodynamic" properties

they may be zero within experimental resolution but this is NOT symmetry forced.

La1 (≈ 0.0 0.25000 ≈ 0.0)

$\frac{1}{4}$ rigorously fulfilled – if broken, it means a different phase

Reminder of symmetry in non-magnetic structures

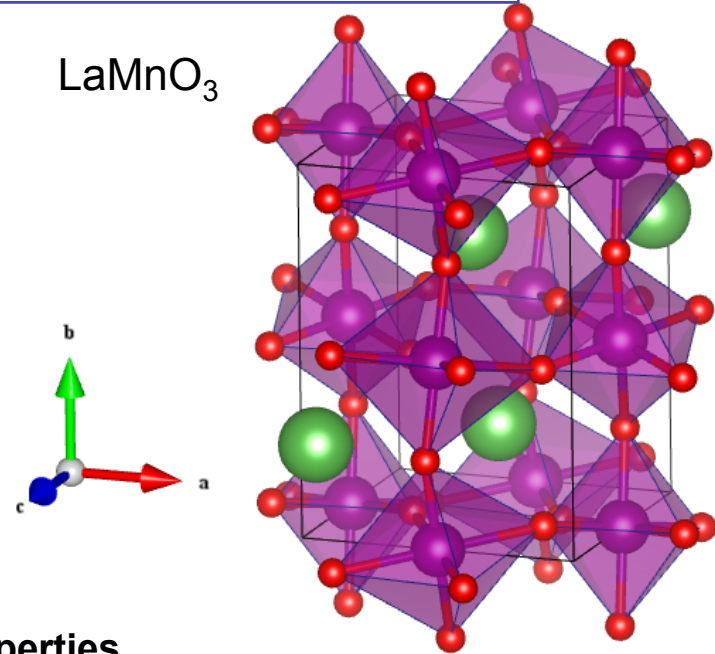
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Whatever microscopic model of atomic forces, if consistently applied, it will yield:
 $F_y(\text{La1}) = 0.000000$ (exact!)

Magnetic Symmetry:

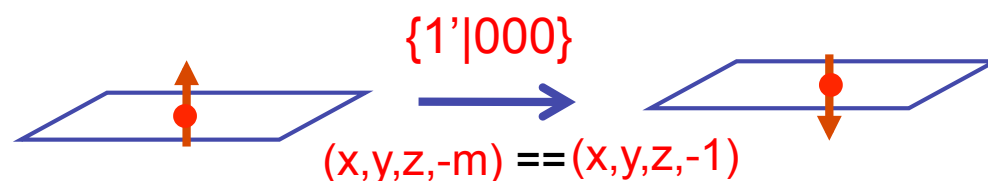
*Symmetry is only detected when it does not exist!
We do not add but subtract symmetry operations !*

Symmetry operation to be considered:

(always present in non-magnetic structures but **ABSENT** in magnetically ordered ones!)

Time inversion/reversal: $\{1'|0,0,0\}$

- Does not change nuclear variables
- Changes sign of ALL atomic magnetic moments



Magnetic structures only have symmetry operations where time reversal $1'$ is combined with other transformations, or is not present at all:

$$\{1'|t\} = \{1'|0,0,0\} \{1|t\}$$

$$\{m'|t\} = \{1'|0,0,0\} \{m|t\}$$

$$\{2'|t\} = \{1'|0,0,0\} \{2|t\}$$

$$\{3'^+|t\} = \{1'|0,0,0\} \{3'^+|t\}, \text{ etc.}$$

But $\{1'|0,0,0\}$ alone is never a symmetry operation of a magn. struct.

All NON-magnetic structures have time inversion symmetry

If all atomic magnetic moments are zero, time inversion is a (trivial) symmetry operation of the structure:

Actual symmetry of the non-magnetic phase:

$$Pnma1' = Pnma + \{1'|000\} \times Pnma \quad (\text{grey group})$$

16 operations:

$$\begin{array}{cccc} (x,y,z,+1) & (-x+1/2,-y,z+1/2,+1) & (-x,y+1/2,-z,+1) & (x+1/2,-y+1/2,-z+1/2,+1) \\ (-x,-y,-z,+1) & (x+1/2,y,-z+1/2,+1) & (x,-y+1/2,z,+1) & (-x+1/2,y+1/2,z+1/2,+1) \end{array}$$

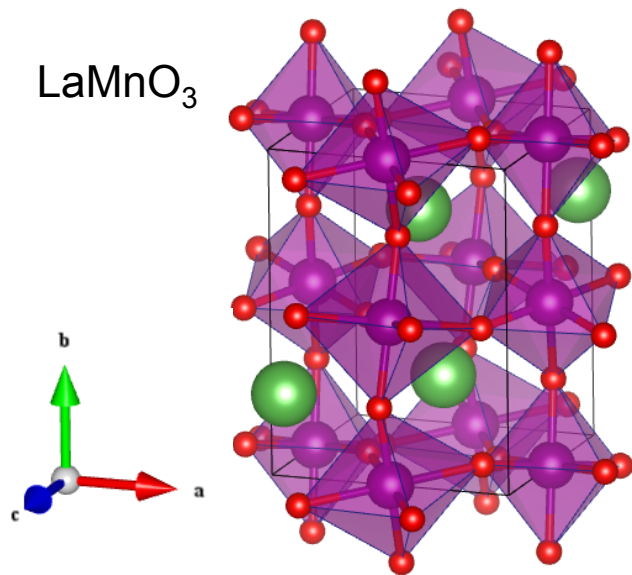
$$\begin{array}{cccc} (x,y,z,-1) & (-x+1/2,-y,z+1/2,-1) & (-x,y+1/2,-z,-1) & (x+1/2,-y+1/2,-z+1/2,-1) \\ (-x,-y,-z,-1) & (x+1/2,y,-z+1/2,-1) & (x,-y+1/2,z,-1) & (-x+1/2,y+1/2,z+1/2,-1) \end{array}$$

Notation: $(x+1/2,-y+1/2,-z+1/2,+1) == \{2x| \frac{1}{2} \frac{1}{2} \frac{1}{2} \} \{R|t\}$
 $(x+1/2,-y+1/2,-z+1/2,-1) == \{2x'| \frac{1}{2} \frac{1}{2} \frac{1}{2} \} \{R'|t\}$

$\{R,\theta|t\} \begin{array}{l} \rightarrow \theta=1 \\ \rightarrow \theta=-1 \end{array}$

magnetic ordering breaks symmetry of time inversion

Magnetic ordered phases:



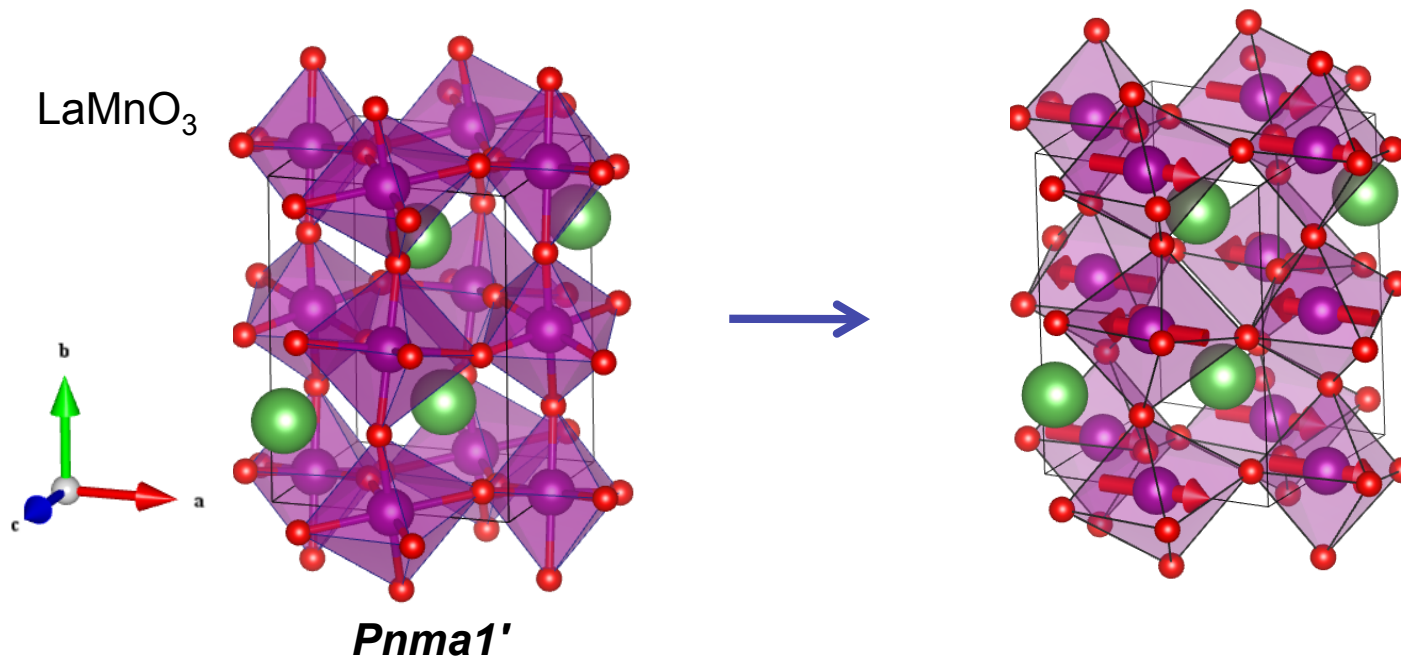
Pnma1'

(x,y,z,+1)	(-x+1/2,-y,z+1/2,+1)	(-x,y+1/2,-z,+1)	(x+1/2,-y+1/2,-z+1/2,+1)
(-x,-y,-z,+1)	(x+1/2,y,-z+1/2,+1)	(x,-y+1/2,z,+1)	(-x+1/2,y+1/2,z+1/2,+1)
(x,y,z,-1)	(-x+1/2,-y,z+1/2,-1)	(-x,y+1/2,-z,-1)	(x+1/2,-y+1/2,-z+1/2,-1)
(-x,-y,-z,-1)	(x+1/2,y,-z+1/2,-1)	(x,-y+1/2,z,-1)	(-x+1/2,y+1/2,z+1/2,-1)

magnetic ordering breaks symmetry of time inversion

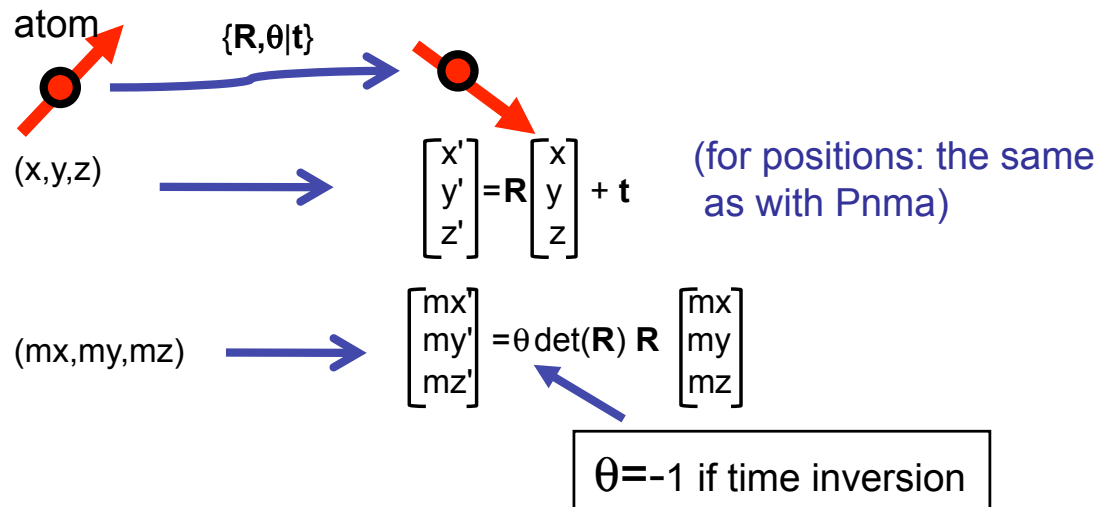
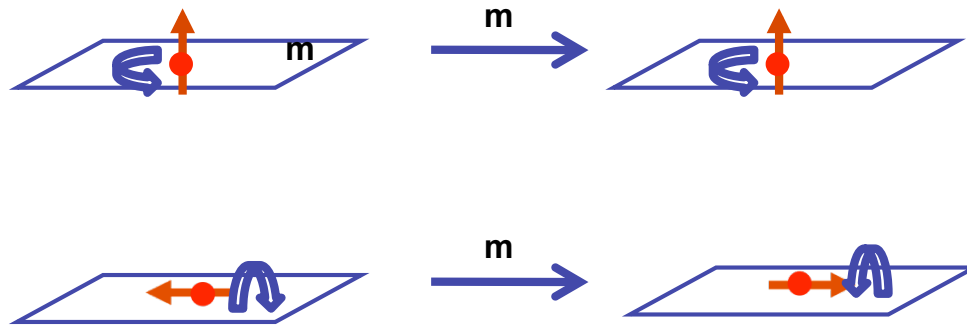
Magnetic ordered phases:

Time inversion $\{1' | 0 0 0\}$ is NOT a symmetry operation of a magnetic phase



For space operations, the magnetic moments transform as pseudovectors or axial vectors:

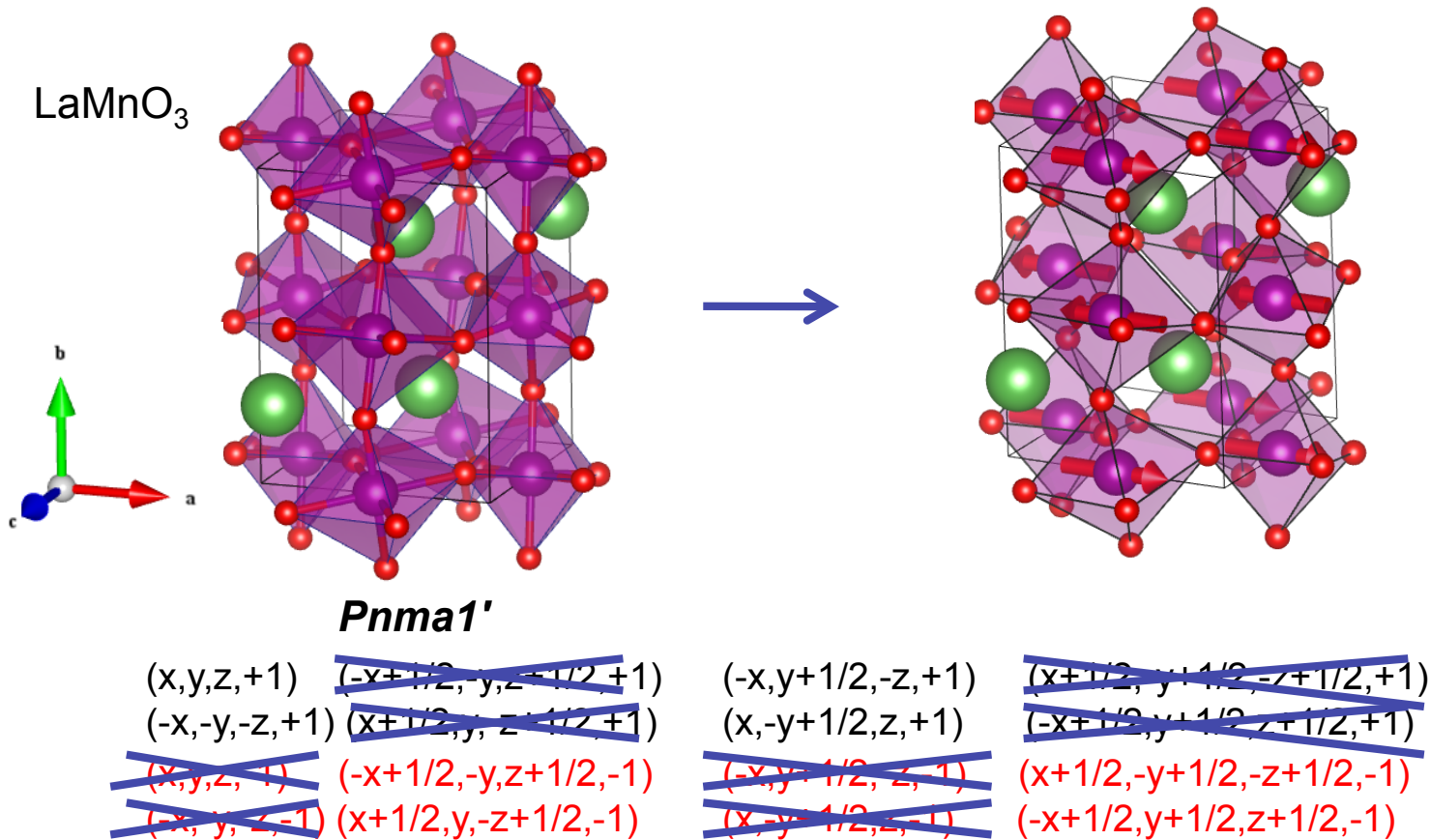
$$T_{\text{axial}}(\mathbf{R}) = \det[\mathbf{R}] \mathbf{R}$$



magnetic ordering breaks symmetry of time inversion

Magnetic ordered phases:

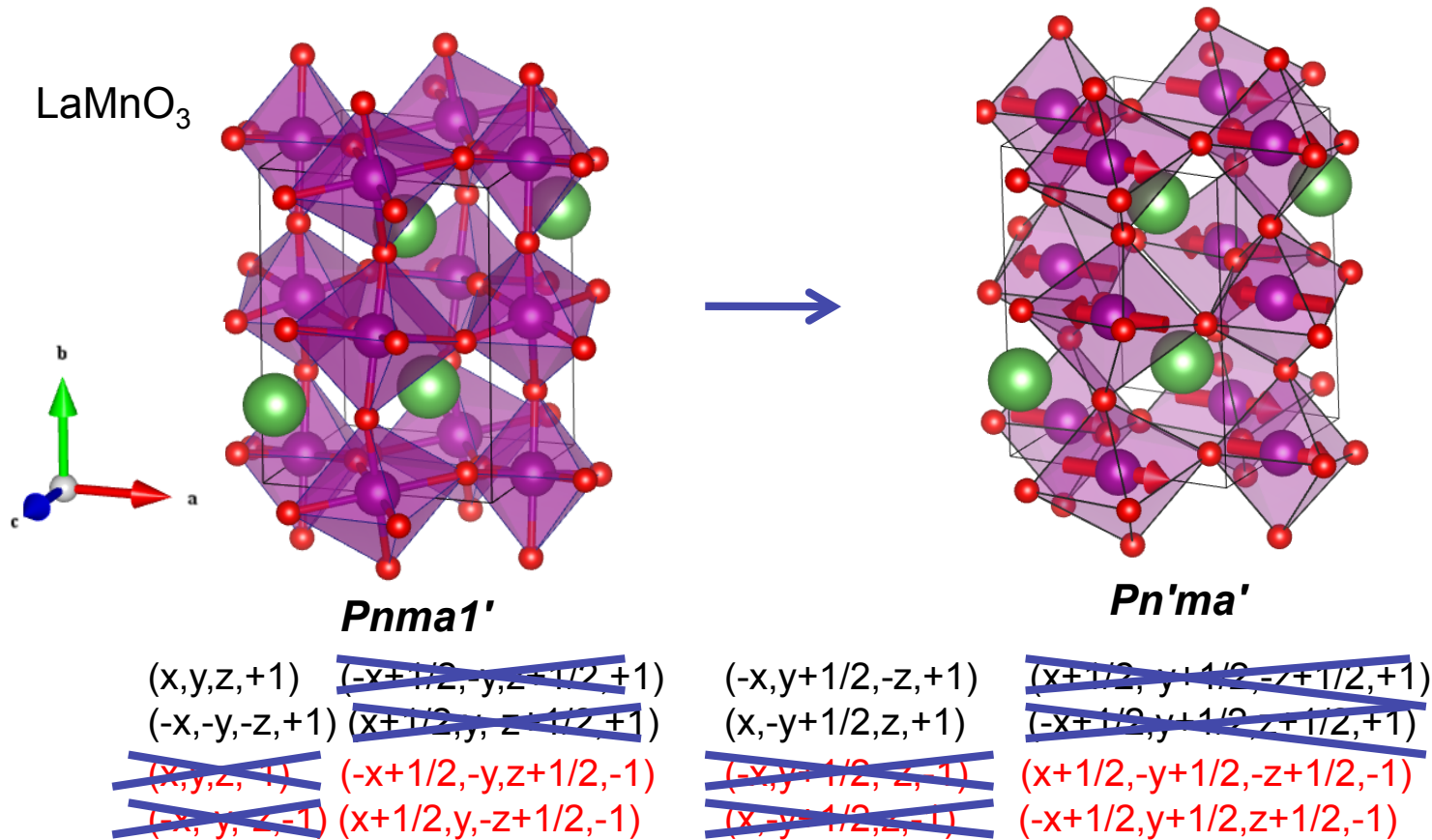
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magnetic ordering breaks symmetry of time inversion

Magnetic ordered phases:

Time inversion $\{1'|0\ 0\ 0\}$ is NOT a symmetry operation of a magnetic phase



MAGNETIC SYMMETRY IN COMMENSURATE CRYSTALS

A symmetry operation fullfills:

- the operation belongs to the set of transformations that keep the **energy invariant**:
 - rotations
 - translations
 - space inversion
 - time reversal
- the system is **undistinguishable** after the transformation

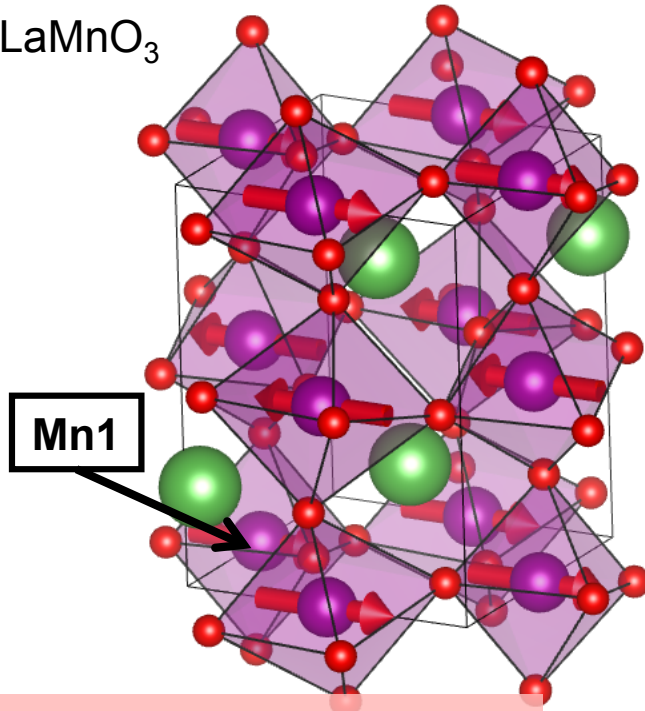
Symmetry operations in commensurate magnetic crystals:

magnetic space group: $\{ \{ \mathbf{R}_i | \mathbf{t}_i \} , \{ \mathbf{R}'_j | \mathbf{t}_j \} \}$

or $\{ \{ \mathbf{R}_i , \theta | \mathbf{t}_i \} \}$ $\theta = +1$ without time reversal
 $\theta = -1$ with time reversal

Description of a magnetic structure in a crystallographic form:

LaMnO₃



Magnetic space Group:
Pn'ma'

Lattice parameters:

5.7461 7.6637 5.5333 90.000 90.000 90.000

Atomic positions of asymmetric unit:

La1 0.05130 0.25000 -0.00950

Mn1 0.00000 0.00000 0.50000

O1 0.48490 0.25000 0.07770

O2 0.30850 0.04080 0.72270

Magnetic moments of the asymmetric unit (μ_B):

Mn1 3.87 0.0 0.0

Pn'ma' :

1 x,y,z,+1

2 -x,y+1/2,-z,+1

3 -x,-y,-z,+1

4 x,-y+1/2,z,+1

5 x+1/2,-y+1/2,-z+1/2,-1

6 -x+1/2,-y,z+1/2,-1

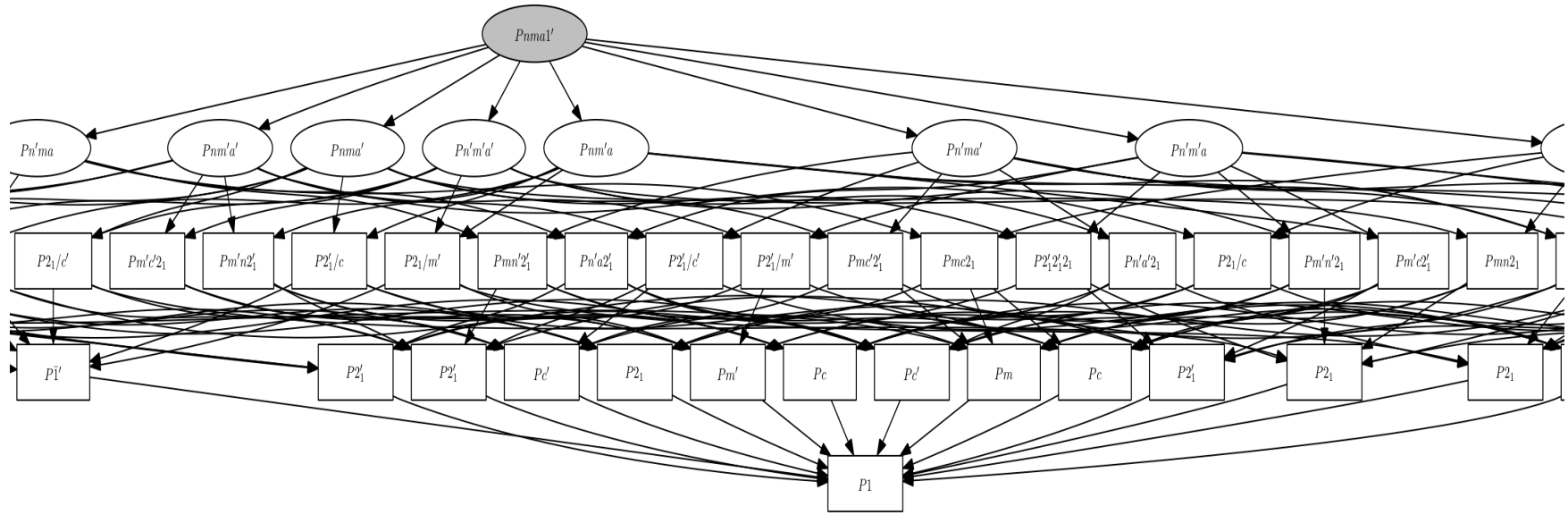
7 -x+1/2,y+1/2,z+1/2,-1

8 x+1/2,y,-z+1/2,-1

Symmetry operations
are relevant both for
positions and moments

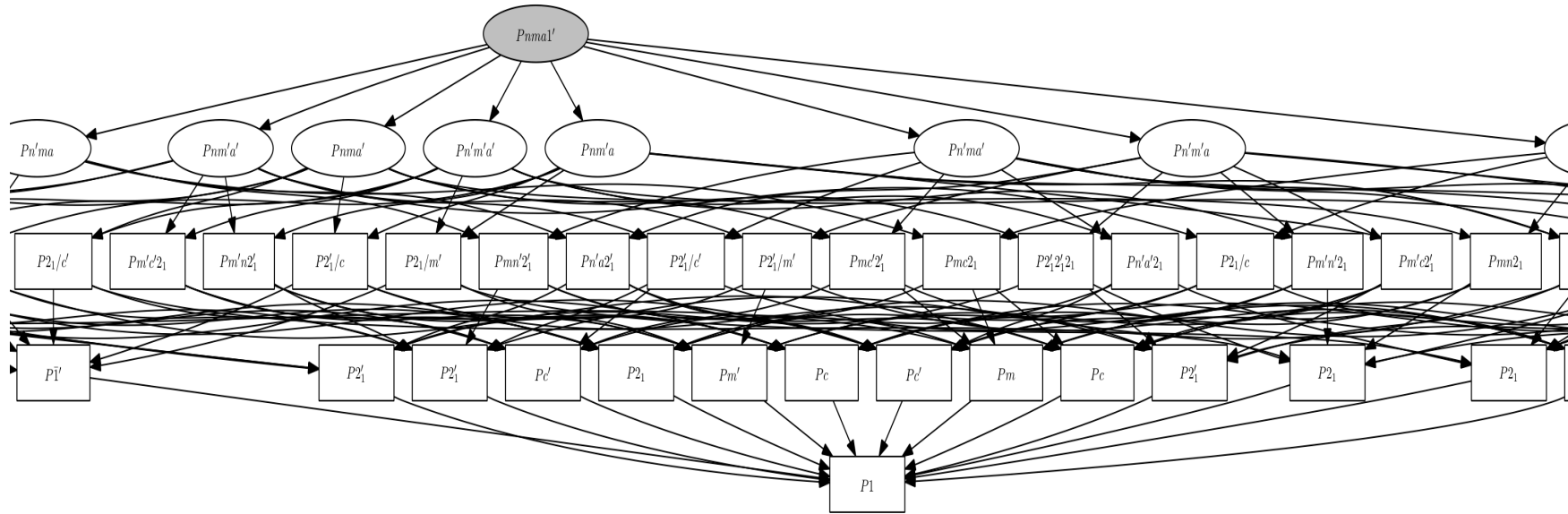
Parent symmetry $Pnma1'$

Possible symmetries for a $k=0$ magnetic ordering:

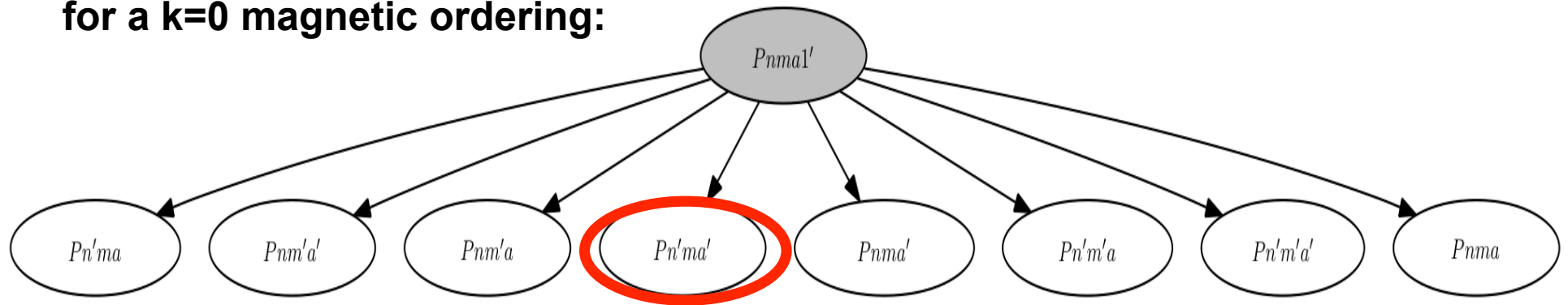


Parent symmetry $Pnma1'$

Possible symmetries for a $k=0$ magnetic ordering:



Possible maximal symmetries for a $k=0$ magnetic ordering:



Magnetic Symmetry and Applications



MGENPOS

General Positions of Magnetic Space Groups

MWYCKPOS

Wyckoff Positions of Magnetic Space Groups

General Positions of the Group $Pn'ma'$ (#62.448)

*For this space group, BNS and OG settings coincide.
Its label in the OG setting is given as: $Pn'ma'$ (#62.8.509)*

N	Standard/Default Setting			
	(x,y,z) form	Matrix form	Geom. interp.	Seitz notation
1	x, y, z, +1 m_x, m_y, m_z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$1 \underline{+1}$	{ 1 0 }
2	-x, y+1/2, -z, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	2 (0, 1/2, 0) 0, y, 0 $\underline{+1}$	{ 2_{010} 0 1/2 0 }
3	-x, -y, -z, +1 m_x, m_y, m_z	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	-1 0, 0, 0 $\underline{+1}$	{ -1 0 }
4	x, -y+1/2, z, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	m x, 1/4, z $\underline{+1}$	{ m_{010} 0 1/2 0 }
5	$x+1/2, -y+1/2, -z+1/2, -1$ $-m_x, m_y, m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (1/2, 0, 0) x, 1/4, 1/4 $\underline{-1}$	{ $2'_{100}$ 1/2 1/2 1/2 }
6	$-x+1/2, -y, z+1/2, -1$ $m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	2 (0, 0, 1/2) 1/4, 0, z $\underline{-1}$	{ $2'_{001}$ 1/2 0 1/2 }
7	$-x+1/2, y+1/2, z+1/2, -1$ $-m_x, m_y, m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (0, 1/2, 1/2) 1/4, y, z $\underline{-1}$	{ m'_{100} 1/2 1/2 1/2 }
8	$x+1/2, y, -z+1/2, -1$ $m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	a x, y, 1/4 $\underline{-1}$	{ m'_{001} 1/2 0 1/2 }

Go to the list of the Wyckoff Positions of the Group $Pn'ma'$ (#62.448)
Go to the Systematic Absences for the Group $Pn'ma'$ (#62.448)

Output of
MGENPOS
in BCS

Magnetic
point group:
 $m'mm'$

Parameters to describe a magnetic structure...

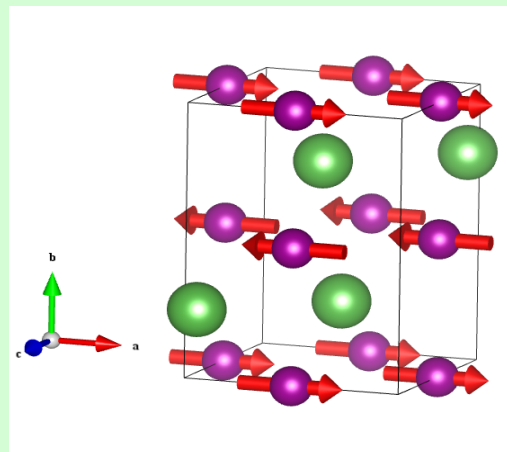
Space Group:
Pn'ma'

Multiplicity	Wyckoff letter	Coordinates
8	d	$(x,y,z m_x, m_y, m_z)$ $(x+1/2, -y+1/2, -z+1/2 -m_x, m_y, m_z)$ $(-x, y+1/2, -z -m_x, m_y, -m_z)$ $(-x+1/2, -y, z+1/2 m_x, m_y, -m_z)$ $(-x, -y, -z m_x, m_y, m_z)$ $(-x+1/2, y+1/2, z+1/2 -m_x, m_y, m_z)$ $(x, -y+1/2, z -m_x, m_y, -m_z)$ $(x+1/2, y, -z+1/2 m_x, m_y, -m_z)$
4	c	$(x, 1/4, z 0, m_y, 0)$ $(x+1/2, 1/4, -z+1/2 0, m_y, 0)$ $(-x, 3/4, -z 0, m_y, 0)$ $(-x+1/2, 3/4, z+1/2 0, m_y, 0)$
4	b	$(0, 0, 1/2 m_x, m_y, m_z)$ $(1/2, 1/2, 0 -m_x, m_y, m_z)$ $(0, 1/2, 1/2 -m_x, m_y, -m_z)$ $(1/2, 0, 0 m_x, m_y, -m_z)$
4	a	$(0, 0, 0 m_x, m_y, m_z)$ $(1/2, 1/2, 1/2 -m_x, m_y, m_z)$ $(0, 1/2, 0 -m_x, m_y, -m_z)$ $(1/2, 0, 1/2 m_x, m_y, -m_z)$

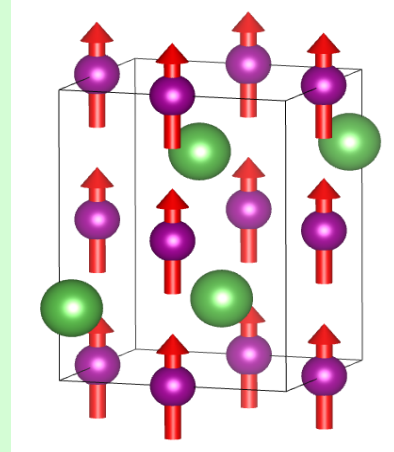
Output of
MWYCKPOS
in BCS

La

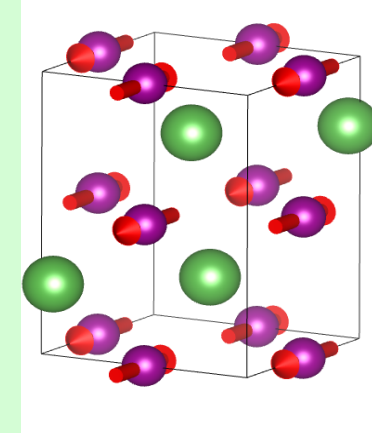
Mn



mode along x (A_x)



mode along y (F_y)
weak ferromagnet



mode along z (G_z)

Description in terms of irrep modes

$$\text{Pn'ma'} \equiv \text{mB}_{2g}$$

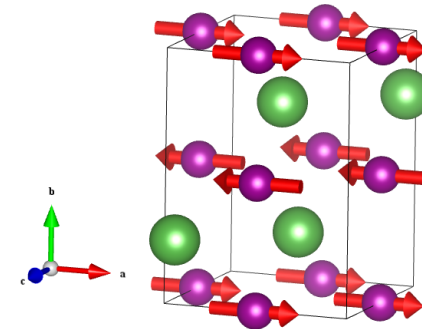
(Irrep = irreducible representation)

Pn'ma'

Character Table

D_{2h} (mmm)	#	1	2_z	2_y	2_x	-1	m_z	m_y	m_x	1'
mA_g	Γ_1^+	1	1	1	1	1	1	1	1	-1
mB_{1g}	Γ_3^+	1	1	-1	-1	1	1	-1	-1	-1
mB_{2g}	Γ_2^+	1	-1	1	-1	1	-1	1	-1	-1
mB_{3g}	Γ_4^+	1	-1	-1	1	1	-1	-1	1	-1
mA_u	Γ_1^-	1	1	1	1	-1	-1	-1	-1	-1
mB_{1u}	Γ_3^-	1	1	-1	-1	-1	-1	1	1	-1
mB_{2u}	Γ_2^-	1	-1	1	-1	-1	1	-1	1	-1
mB_{3u}	Γ_4^-	1	-1	-1	1	-1	1	1	-1	-1

$$\begin{matrix} 2_z' & 2_y' & 2_x' & -1' & m_z' & m_y' & m_x' \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{matrix}$$



one mB_{2g} spin mode
(or basis function)

Building up the model of the magnetic structure of LaMnO₃ using STRCONVERT

1. Upload the cif file of LaMnO₃ in STRCONVERT. Change to magnetic option. Transform to P1 to produce the whole set of atomic positions within the unit cell. Introduce magnetic moments along x of the four listed Mn atoms according to the sign relations: 1,-1,-1,1. Use findsym to find the MSG, and the description using this MSG. Visualize the magnetic structure with MVISUALIZE. With the back button come back in the STRCONVERT to the page in P1, and introduce a non-zero component my at the single symmetry-independent Mn atom, and transform again to P1 to observe that the resulting my values for 4 Mn atoms within the unit cell have the same sign (expected weak FM along y). Create an mcif file of the structure with STRCONVERT, open it with a text editor and localize the different data items: unit cell, atomic positions, symmetry operations. Download the mcif file in VESTA and visualize the structure with this tool. (file required: *1.LaMnO3_parent.cif*)

Structure Utilities	
CELLTRAN	Transform Unit Cells
STRAIN	Strain Tensor Calculation
WPASSIGN	Assignment of Wyckoff Positions
TRANSTRU	Transform structures.
SETSTRU ⚠	Alternative Settings for a given Crystal Structure
EQUIVSTRU	Equivalent Descriptions for a given Crystal Structure
→ STRCONVERT	Convert & Edit Structure Data (supports the CIF, mCIF, VESTA, VASP formats -- with magnetic information where available)

Structure Data Converter & Editor

Please submit a structure file:

No file selected.

[Supported file formats: [CIF](#), [mCIF](#), [VESTA](#), [VASP](#)]

Symmetry

Magnetic Space Group # (BNS): (P_bmn2_1) Type: IV

Lattice Parameters

a: Å b: Å c: Å

α ° β : ° γ : °

Symmetry Operations [\[Show/Hide\]](#)

Recognized formats:

1. x,y,z mx,my,mz +1

x,y,z mx,my,mz +1

x,y,z

x,y,z,+1

1 'x, y, z'

1 x,y,z

```
x, y, z, +1
-x+1/4, -y, z+1/2, +1
x, -y+1/2, z, +1
-x+1/4, y+1/2, z+1/2, +1
x+1/2, y, z, -1
-x+3/4, -y, z+1/2, -1
x+1/2, -y+1/2, z, -1
-x+3/4, y+1/2, z+1/2, -1
```

Update the symmetry operators with the above

Symmetry operations have been parsed from the file/form

Structure Data Converter & Editor

Atomic Positions & Magnetic Moments

Switch to the treatment of the vectors as:

	Label	Element	x	y	z	Occ.	m_x	m_y	m_z
<input type="checkbox"/>	Ho	Ho	0.04195	0.25000	0.98250	1.00000	0.00000	0.00000	0.00000
<input type="checkbox"/>	Ho_1	Ho	0.95805	0.75000	0.01750	1.00000	0.00000	0.00000	0.00000
<input type="checkbox"/>	Mn	Mn	0.00000	0.00000	0.50000	1.00000	3.87000	0.00000	0.00000
<input type="checkbox"/>	O1	O	0.23110	0.25000	0.11130	1.00000	0.00000	0.00000	0.00000
<input type="checkbox"/>	O1_1	O	0.76890	0.75000	0.88870	1.00000	0.00000	0.00000	0.00000
<input type="checkbox"/>	O2	O	0.16405	0.05340	0.70130	1.00000	0.00000	0.00000	0.00000
<input type="checkbox"/>	O2_1	O	0.83595	0.55340	0.29870	1.00000	0.00000	0.00000	0.00000

 atoms more.

||

||

||

||

Longest Arrow size: [for VESTA format export: Å (Default: $\min(a,b,c)/4$)]
[for Jmol visualize: a proportional coefficient]

a CIF-type file can be produced:

These files permit the different alternative models to be analyzed, refined, shown graphically, transported to ab-initio codes etc., with programs as **ISODISTORT, **JANA2006**, **STRCONVERT**, etc. A controlled descent to lower symmetries is also possible.**

```
_magnetic_space_group_BNS_number 31.129
_magnetic_space_group_BNS_name "P_bmn2_1"
_magnetic_cell_length_a 11.67080
_magnetic_cell_length_b 7.36060
_magnetic_cell_length_c 5.25720
_magnetic_cell_angle_alpha 90.00
_magnetic_cell_angle_beta 90.00
_magnetic_cell_angle_gamma 90.00

loop_
_magnetic_space_group_symop_id
_magnetic_space_group_symop_operation_xyz
_magnetic_space_group_symop_operation_mxmy mz
_magnetic_space_group_symop_operation_timereversal
1 x,y,z mx,my,mz +1
2 -x+1/4,-y,z+1/2 -mx,-my,mz +1
3 x,-y+1/2,z -mx,my,-mz +1
4 -x+1/4,y+1/2,z+1/2 mx,-my,-mz +1
5 x+1/2,y,z -mx,-my,-mz -1
6 -x+3/4,-y,z+1/2 mx,my,-mz -1
7 x+1/2,-y+1/2,z mx,-my,mz -1
8 -x+3/4,y+1/2,z+1/2 -mx,my,mz -1

loop_
_magnetic_atom_site_label
_magnetic_atom_site_type_symbol
_magnetic_atom_site_fract_x
_magnetic_atom_site_fract_y
_magnetic_atom_site_fract_z
Ho Ho 0.04195 0.25000 0.98250
Ho_1 Ho 0.95805 0.75000 0.01750
Mn Mn 0.00000 0.00000 0.50000
01 0 0.23110 0.25000 0.11130
01_1 0 0.7689 0.75000 0.88870
02 0 0.16405 0.05340 0.70130
02_1 0 0.83595 0.55340 0.29870

loop_
_magnetic_atom_site_moment_label
_magnetic_atom_site_moment_crystalaxis_mx
_magnetic_atom_site_moment_crystalaxis_my
_magnetic_atom_site_moment_crystalaxis_mz
Mn 3.87 0.00000 0.00000
```

<http://stokes.byu.edu/iso/isotropy.php>

ISOTROPY Software Suite

Harold T. Stokes, Dorian M. Hatch, and Branton J. Campbell, Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84606, USA, stokes@byu.edu

Description: The ISOTROPY software suite is a collection of software which applies group theoretical methods to the analysis of phase transitions in crystalline solids.

How to cite: ISOTROPY Software Suite, iso.byu.edu.

References and Resources

Isotropy subgroups and distortions

- **ISODISTORT:** Explore and visualize distortions of crystalline structures. Possible distortions include atomic displacements, atomic ordering, strain, and magnetic moments.
- **New! ISOSUBGROUP:** Interactive program using user-friendly interface to list isotropy subgroups.
- **ISOTROPY:** Interactive program using command lines to explore isotropy subgroups and their associated distortions.
- **SMODES:** Find the displacement modes in a crystal which brings the dynamical matrix to block-diagonal form, with the smallest possible blocks.
- **FROZSL:** Calculate phonon frequencies and displacement modes using the method of frozen phonons.

Space groups and irreducible representations

- **ISOCIF:** Create or modify CIF files.
- **FINDSYM:** Identify the space group of a crystal, given the positions of the atoms in a unit cell.
- **ISO-IR:** Tables of Irreducible Representations. The 2011 version of IR matrices.
- **ISO-MAG:** Tables of magnetic space groups, both in human-readable and computer-readable forms.

Superspace Groups

- **ISO(3+d)D:** (3+d)-Dimensional Superspace Groups for $d=1,2,3$
- **ISO(3+1)D:** Isotropy Subgroups for Incommensurately Modulated Distortions in Crystalline Solids: A Complete List for One-Dimensional Modulations
- **FINDSSG:** Identify the superspace group symmetry given a list of symmetry operators.
- **TRANSFORMSSG:** Transform a superspace group to a new setting.

Phase Transitions

- **COPL:** Find a complete list of order parameters for a phase transition, given the space-group symmetries of the parent and subgroup phases.
- **INVARIANTS:** Generate invariant polynomials of the components of order parameters.
- **COMSUBS:** Find common subgroups of two structures in a reconstructive phase transition

Other CIF (and mCIF) file editor

ISOCIF

Version 2.3.12, Oct 2015

Harold T. Stokes and Branton J. Campbell, Department of Physics and Astronomy, Brigham Young University, Provo, Utah, 84602, USA, stokes@byu.edu

Description: ISOCIF is a tool for creating and modifying CIF structure files.

[Help](#)

[Version History](#)

How to cite ISOCIF: ISOTROPY Software Suite, iso.byu.edu.

Create new CIF file

Modify existing CIF file: No se ha selecci..

Read old mCIF format

General Positions of the Group $Pn'ma'$ (#62.448)

For this space group, BNS and OG settings coincide.
Its label in the OG setting is given as: $Pn'ma'$ (#62.8.509)

N	Standard/Default Setting			
	(x,y,z) form	Matrix form	Geom. interp.	Seitz notation
1	x, y, z, +1 m_x, m_y, m_z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$1 \underline{+1}$	{ 1 0 }
2	-x, y+1/2, -z, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	2 (0, 1/2, 0) 0, y, 0 $\underline{+1}$	{ 2 ₀₁₀ 0 1/2 0 }
3	-x, -y, -z, +1 m_x, m_y, m_z	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	-1 0, 0, 0 $\underline{+1}$	{ -1 0 }
4	x, -y+1/2, z, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	m x, 1/4, z $\underline{+1}$	{ m ₀₁₀ 0 1/2 0 }
5	x+1/2, -y+1/2, -z+1/2, -1 $-m_x, m_y, m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	2 (1/2, 0, 0) x, 1/4, 1/4 $\underline{-1}$	{ 2' ₁₀₀ 1/2 1/2 1/2 }
6	-x+1/2, -y, z+1/2, -1 $m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	2 (0, 0, 1/2) 1/4, 0, z $\underline{-1}$	{ 2' ₀₀₁ 1/2 0 1/2 }
7	-x+1/2, y+1/2, z+1/2, -1 $-m_x, m_y, m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (0, 1/2, 1/2) 1/4, y, z $\underline{-1}$	{ m' ₁₀₀ 1/2 1/2 1/2 }
8	x+1/2, y, -z+1/2, -1 $m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	a x, y, 1/4 $\underline{-1}$	{ m' ₀₀₁ 1/2 0 1/2 }

Go to the list of the Wyckoff Positions of the Group $Pn'ma'$ (#62.448)
Go to the Systematic Absences for the Group $Pn'ma'$ (#62.448)

Output of
MGENPOS

Magnetic
point group:
 $m'mm'$

$$Pn'ma' = P12_1/m1 + \{2'_{100}|1/2, 1/2, 1/2\} P12_1/m1$$

Types of magnetic space groups:

(for a commensurate magnetic structure resulting from a paramagnetic phase having a grey magnetic group $G1'$)

Time inversion $\{1' | 0 0 0\}$ is NOT a symmetry operation of magnetic structure, but combined with a translation it can be...

F subgroup of G
 $F \leq G$

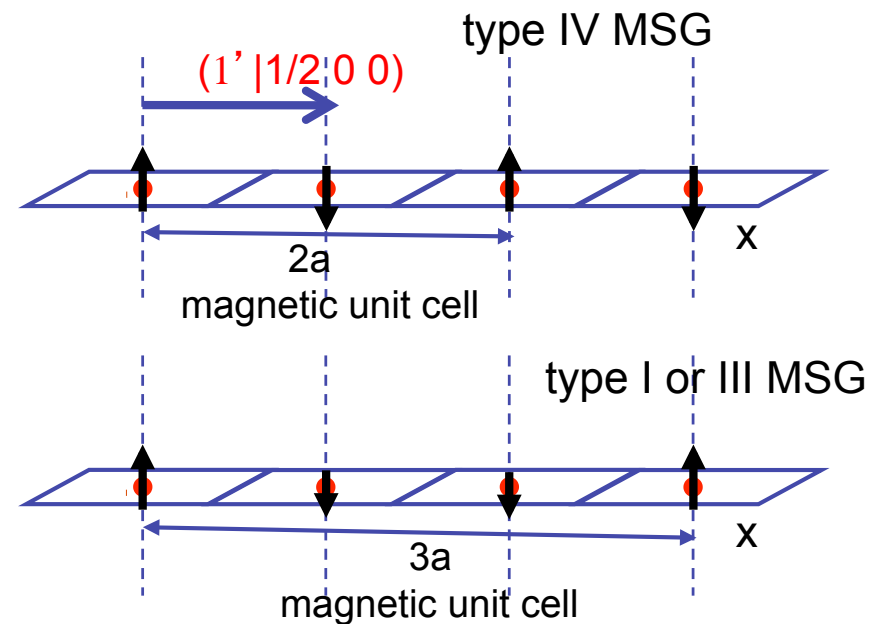
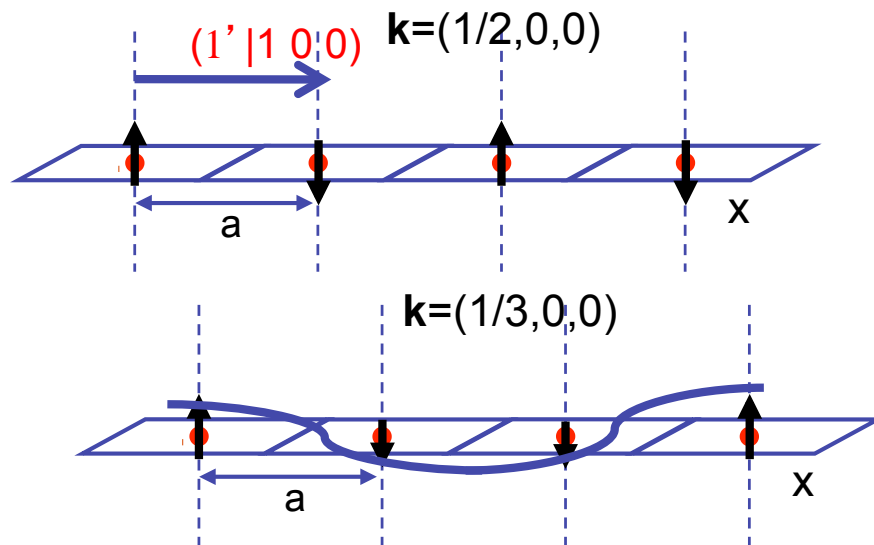
	magn. space group:	magn. point groups:	nuclear space group: (space group)
Type I some may allow ferromagnetic order	F	P_F	F
Type III some may allow ferromagnetic order	F + {R' t}F	black and white group P_F + R'P_F	F + {R t}F = H
Type IV antiferromagnetic order (ferromagnetism not allowed)	F + {1' t}F	grey group P_F + 1' P_F	(lattice duplicated) F + {1 t}F = H

antittranslation / anticentering

(Type II are the grey groups)

Type of MSG depends on the propagation vector of the magnetic ordering:

- Most magn. orderings are 1k-magnetic structures.
- 1k-magnetic structures: moment changes from one unit cell to another according to a single wave vector or propagation vector \mathbf{k} .
- Phase factor for unit cell \mathbf{T} : $\exp(-i2\pi\mathbf{k}\cdot\mathbf{T})$
- The lattice translations such that $\exp(-i2\pi\mathbf{k}\cdot\mathbf{T})=1$ define the lattice maintained by the magnetic structure.
- **The lattice translations such that $\exp(-i2\pi\mathbf{k}\cdot\mathbf{T})= -1$, are kept as antitranslations (type IV MSG). Only occur if $n\mathbf{k}=\text{recip. lattice vector with } n=\text{even}$**



multiple k structures: analogous situation ...

General Positions of the Group P_bmn2_1 (#31.129) [BNS setting]

To display the general positions in the OG setting, please follow this link: [P_{2b}mn2₁ \(#31.6.217\) \[Transformation matrix\]](#)

Translation lattice generators: (1|1,0,0), (1|0,1,0), (1|0,0,1), (1|0,0,0)

Black-and-white lattice generators: (1|1,0,0), (1|0,1,0), (1|0,0,1), (**1'|0,1/2,0**)

N	Standard/Default Setting			
	(x,y,z) form	Matrix form	Geom. interp.	Seitz notation
1	x, y, z, +1 m_x, m_y, m_z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	1 <u>+1</u>	{ 1 0 }
2	-x+1/2, -y, z+1/2, +1 $-m_x, -m_y, m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	2 (0,0,1/2) 1/4,0,z <u>+1</u>	{ 2 ₀₀₁ 1/2 0 1/2 }
3	-x, y, z, +1 $m_x, -m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	m 0,y,z <u>+1</u>	{ m ₁₀₀ 0 }
4	x+1/2, -y, z+1/2, +1 $-m_x, m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$	n (1/2,0,1/2) x,0,z <u>+1</u>	{ m ₀₁₀ 1/2 0 1/2 }
5	x, y+1/2, z, -1 $-m_x, -m_y, -m_z$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ *	t (0,1/2,0) <u>-1</u>	{ 1' 0 1/2 0 }
6	-x+1/2, -y+1/2, z+1/2, -1 $m_x, m_y, -m_z$	$\begin{pmatrix} -1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$ *	2 (0,0,1/2) 1/4,1/4,z <u>-1</u>	{ 2' ₀₀₁ 1/2 1/2 1/2 }
7	-x, y+1/2, z, -1 $-m_x, m_y, m_z$	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ *	b 0,y,z <u>-1</u>	{ m' ₁₀₀ 0 1/2 0 }
8	x+1/2, -y+1/2, z+1/2, -1 $m_x, -m_y, m_z$	$\begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$ *	n (1/2,0,1/2) x,1/4,z <u>-1</u>	{ m' ₀₁₀ 1/2 1/2 1/2 }

Output of
MGENPOS

Example
of type IV
MSG

Propagation
vector $k \neq 0$

$$P_bmn2_1 = Pmn2_1 + \{1'|0,1/2,0\} Pmn2_1$$

Tables of magnetic space groups ("standard" settings)

1.- E-book: D.B. Litvin: "Magnetic space groups" (Electronic Book)

Litvin DB. 2013. *Magnetic Group Tables: 1-, 2- and 3-Dimensional Magnetic Subperiodic Groups and Magnetic Space Groups*. Chester, UK: Int. Union Crystallogr. <http://www.iucr.org/publ/978-0-9553602-2-0>

(listing using only OG setting)

2.- Computer readable listing:

ISOTROPY webpage: <http://stokes.byu.edu/iso/magneticspacegroups.html>

H.T. Stokes and B.J. Campbell

(listing using BNS and OG settings)

3.- Bilbao crystallographic server (www.cryst.ehu.es)

Magnetic Symmetry and Applications

MGENPOS

General Positions of Magnetic Space Groups

MWYCKPOS





Wyckoff Positions of Magnetic Space Groups

(listings using BNS and OG settings)

Here we will always use BNS settings for the MSGs




Magnetic symmetry tools and applications in the BCS :

Magnetic Symmetry and Applications

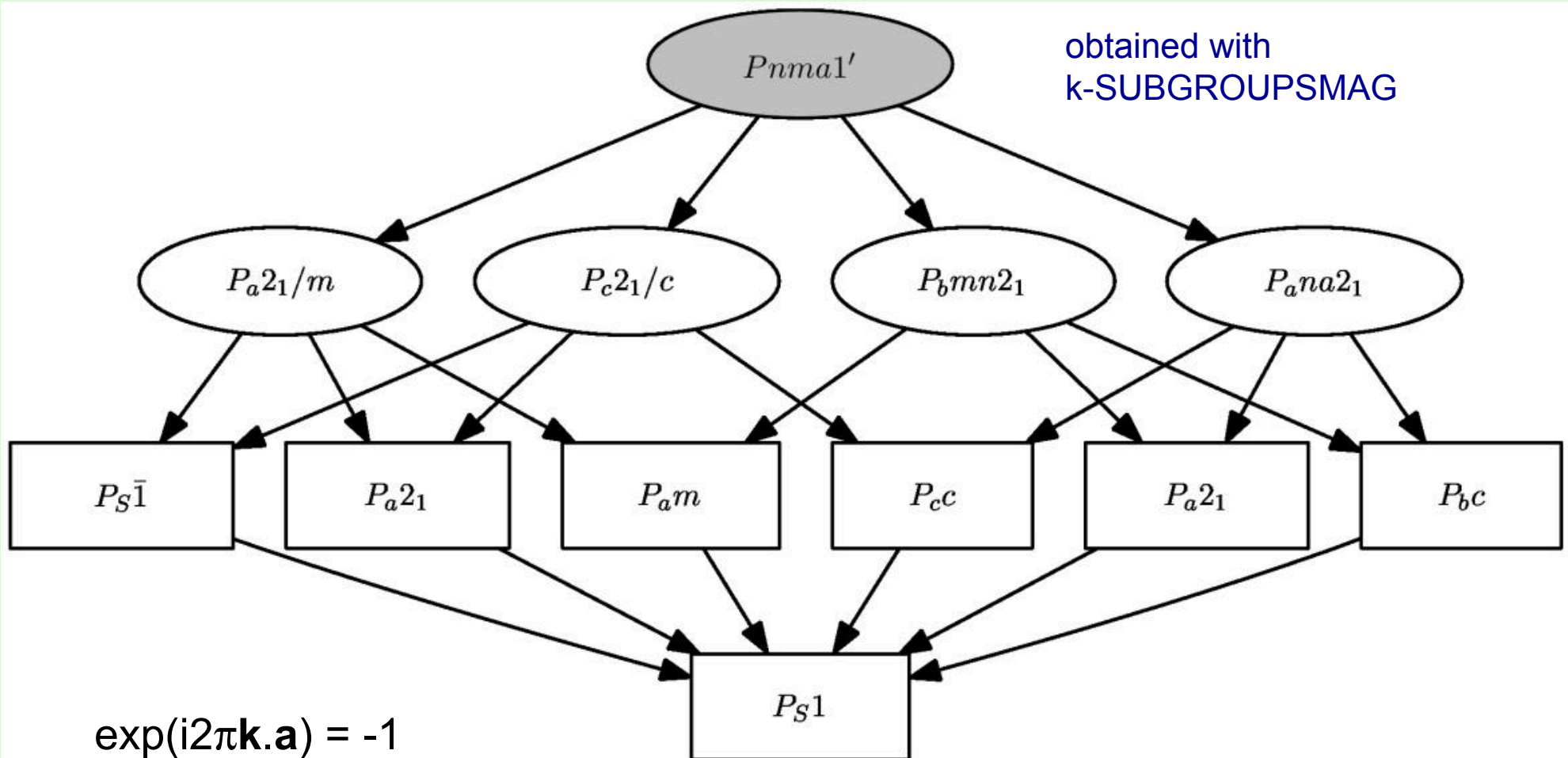
MGENPOS	General Positions of Magnetic Space Groups
MWYCKPOS	Wyckoff Positions of Magnetic Space Groups
MAGNEXT	Extinction Rules of Magnetic Space Groups
IDENTIFY MAGNETIC GROUP	Identification of a Magnetic Space Group from a set of generators in an arbitrary setting
MPOINT 	Magnetic Point Group Tables
MAXMAGN	Maximal magnetic space groups for a given a propagation vector and resulting magnetic structural models
MAGMODELIZE	Magnetic structure models for any given magnetic symmetry
k-SUBGROUPSMAG	Magnetic subgroups consistent with some given propagation vector(s) or a supercell
MAGNDATA 	A collection of magnetic structures with transportable cif-type files
MVISUALIZE 	3D Visualization of magnetic structures with Jmol
MTENSOR 	Symmetry-adapted form of magnetic crystal tensors

Obtaining all possible magnetic symmetries for a magnetic phase with propagation vector $(1/2,0,0)$ and parent space group Pnma

2. Using [k-SUBGROUPSMAG](#) we obtain the k-maximal subgroups for the parent space group Pnma for a propagation vector $k=(1/2,0,0)$.

Magnetic Symmetry and Applications	
MGENPOS	General Positions of Magnetic Space Groups
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MAGNDATA 	A collection of magnetic structures with transportable cif-type files

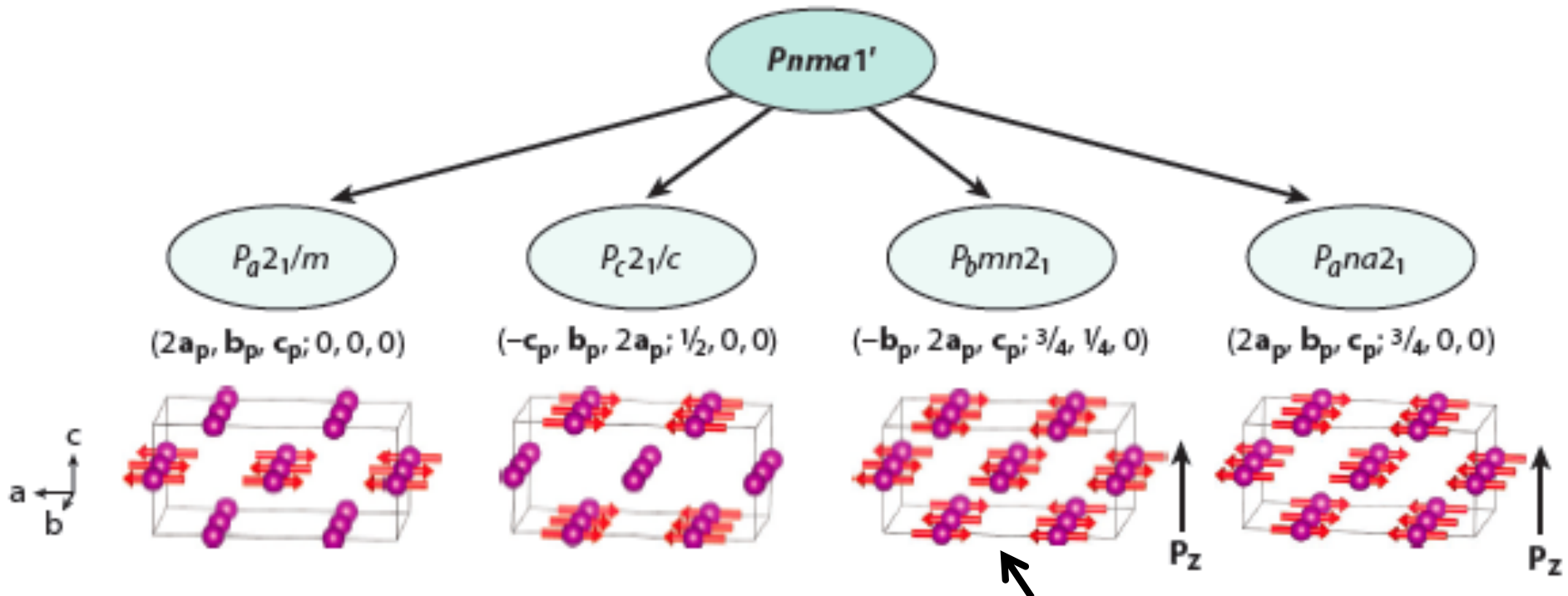
Possible magnetic symmetries for a magnetic phase with propagation vector $(1/2,0,0)$ and parent space group $Pnma$



**Symmetry operation $\{1'|1/2,0,0\}$ is present in any case: all MSGs are type IV
(magnetic cell = $(2\mathbf{a}_p, \mathbf{b}_p, \mathbf{c}_p)$)**

HoMnO₃ An Inevitable multiferroic...

parent space group: Pnma, $k=(1/2,0,0)$

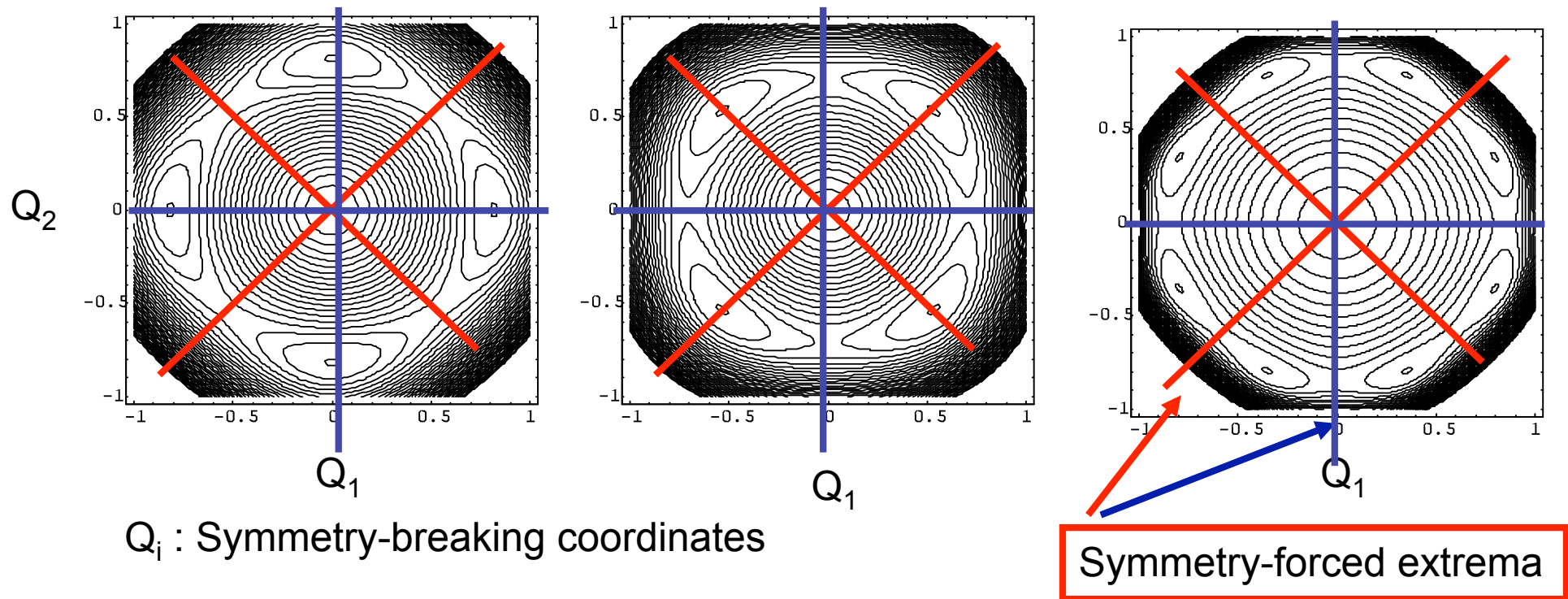


graphic models are depicted assuming collinearity along x (my and mz are symmetry allowed)

Structure reported in 2001, but authors unaware of its multiferroic character

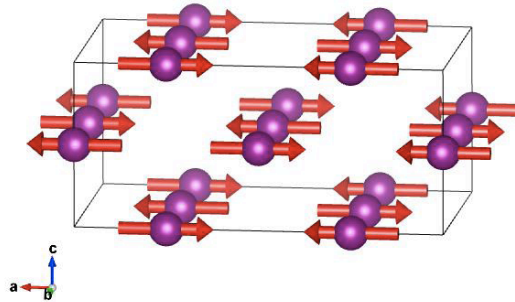
Why the (magnetic) order parameter usually takes “special” directions of higher symmetry in the irrep space?

Domains/variants: symmetry related configurations around a higher-symmetry configuration



HoMnO₃

unit cell: $2a_p, b_p, c_p$



Equivalent to the use of space group **Pnm2₁(31)** with **half cell along a**:

Atomic positions of asymmetric unit:

Ho1	4a	0.04195	0.25000	0.98250
Ho2	4a	0.95805	0.75000	0.01750
Mn1	8b	0.00000	0.00000	0.50000
O1	4a	0.23110	0.25000	0.11130
O12	4a	0.76890	0.75000	0.88870
O2	8b	0.16405	0.05340	0.70130
O22	8b	0.83595	0.55340	0.29870

General position:
x, y, z not restricted
by symmetry!

Magnetic space group: **P_bmn2₁** (31.129)

in non-standard setting.

to transform to conventional setting :

$(-b, a, c; 3/8, 1/4, 0)$

WP	+ (1' 1/2 0 0)
8b	$(x, y, z m_x, m_y, m_z), (-x+1/4, -y, z+1/2 -m_x, -m_y, m_z),$ $(x, -y+1/2, z -m_x, m_y, -m_z), (-x+1/4, y+1/2, z+1/2 m_x, -m_y, -m_z)$
4a	$(x, 1/4, z 0, m_y, 0), (-x+1/4, 3/4, z+1/2 0, -m_y, 0)$

Magnetic moments of the asymmetric unit (μ_B):

Mn1 **3.87** ≈ 0.0 ≈ 0.0



Split independent
positions in the lower
symmetry

zero values are not
symmetry "protected"

Magnetic Point Group: **mm21'**

Checking the symmetry operations present in the magnetic structure of HoMnO_3 .

3. Upload the mcif file of HoMnO_3 in [STRCONVERT](#). Among the listed symmetry operations identify the anticyclerating operation $\{1'|1/2,0,0\}$. Identify also in the list the operations $\{2_z|3/4,0,1/2\}$ and $\{2_z'|1/4,0,1/2\}$. Copy/paste the list of symmetry operations and introduce them in the program "[IDENTIFY MAGNETIC GROUP](#)" and check the MSG of the structure.
(file required: *2.HoMnO3.mcif*)

Magnetic Symmetry and Applications	
MGENPOS	General Positions of Magnetic Space Groups
MWYCKPOS	Wyckoff Positions of Magnetic Space Groups
MAGNEXT	Extinction Rules of Magnetic Space Groups
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MPOINT 	Magnetic Point Group Tables
MAXMAGN	Maximal magnetic space groups for a given a propagation vector and resulting magnetic structural models

Subgroup of $Pnma1'$ in a basis: $2a_p, b_p, c_p$

- 1 $x, y, z, +1$
- 2 $-x+3/4, -y, z+1/2, +1$
- 3 $x, -y+1/2, z, +1$
- 4 $-x+3/4, y+1/2, z+1/2, +1$

centering and “antcentering” translations:

- 1 $x, y, z, +1$
- 2 $x+1/2, y, z, -1$

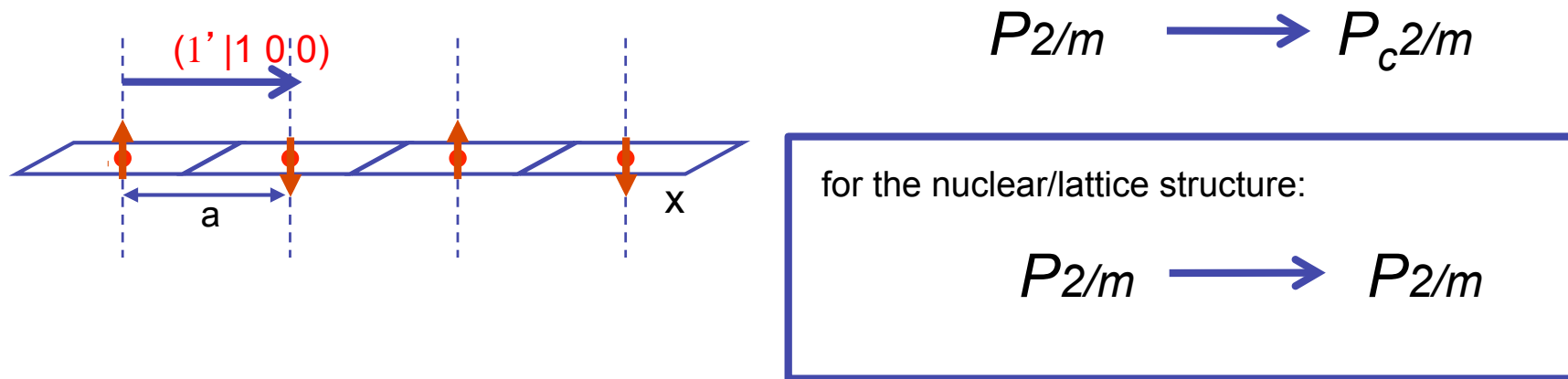
$$Pnma1' = P_a nm2_1 + (1'|000)P_a nm2_1 + (-1|000) P_a nm2_1 + (-1'|000) P_a nm2_1$$

It is the MSG $P_b mn2_1$ (31.129) in a non-standard basis
Transformation to its standard setting: $(-b, a, c; 3/8, 1/4, 0)$

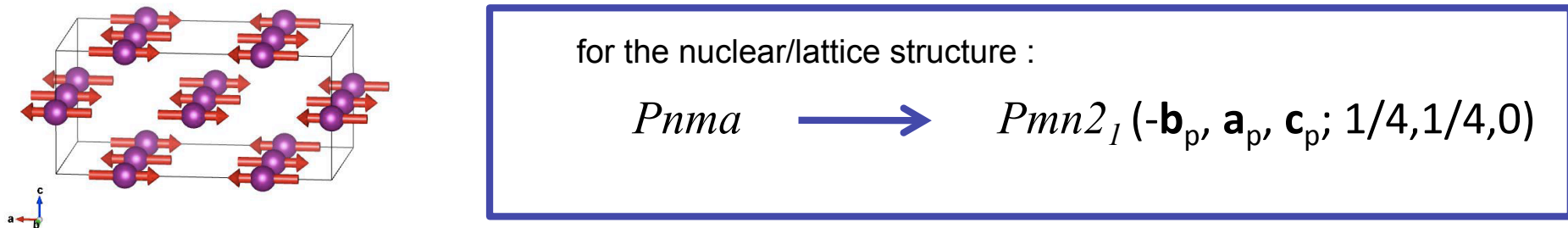
Consequences of symmetry

Effect of the magnetic ordering on the nuclear/lattice structure:

- case 1: no symmetry break for “nuclear structure”



- case 2: symmetry break for “nuclear structure”



Consequences of symmetry

Von Neumann principle:

- all variables/parameters/degrees of freedom compatible with the symmetry can be present in the total distortion
- Tensor crystal properties are constrained by the point group symmetry of the crystal.
- Reversely: any tensor property allowed by the point group symmetry can exist (large or small, but not forced to be zero)

Consequences of symmetry

Symmetry-constrained tensor properties:

TABLE 1.7. HEESCH-SHUBNIKOV POINT GROUPS OF CRYSTALS WHICH MAY EXHIBIT FERROMAGNETISM AND FERROELECTRICITY
(Ascher, 1970) (from 122 point groups)

Point group	M and P
<i>2, m'm'2, 3, 3m', 4, 4m'm', 6, 6m'm'</i>	M <i>z</i> P <i>z</i>
<i>m'm2'</i>	M <i>y</i> P <i>z</i>
<i>2'</i>	M ⊥ <i>z</i> P <i>z</i>
<i>m</i>	M <i>z</i> P ⊥ <i>z</i>
<i>m'</i>	M ⊥ <i>z</i> P ⊥ <i>z</i>
1	No restriction

A. P. Cracknell, "Magnetism in crystalline materials" 1975

Consequences of symmetry

Symmetry-constrained
tensor properties:

A. P. Cracknell,
B. "Magnetism in crystalline materials" 1975

TABLE 5.4. THE MAGNETOELECTRIC TENSOR

Magnetic point group	Q_{ij}
$1, I'$	$\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}$
$2, m', 2/m'$	$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}$
$2', m, 2'/m$	$\begin{bmatrix} 0 & 0 & Q_{13} \\ 0 & 0 & Q_{23} \\ Q_{31} & Q_{32} & 0 \end{bmatrix}$
$222, m'm'2, m'm'm'$	$\begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}$
$22'2', mm2, (m'm2'), m'mm$	$\begin{bmatrix} 0 & Q_{12} & 0 \\ Q_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$4, \bar{4}', 4/m', 3, \bar{3}', 6, \bar{6}', 6/m'$	$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ -Q_{12} & Q_{11} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}$
.....

MTENSOR: Tensor calculation for Magnetic Point Groups

Tensor calculation for Magnetic Point Groups

MTENSOR provides the symmetry-adapted form of tensor properties for any magnetic point (or space) group. On the one hand, a point or space group must be selected, either in standard setting or in a non-standard setting defined by means of a transformation matrix to the standard setting or a set of generators of the magnetic point group. On the other hand, a tensor must be defined by the user or selected from the lists of known equilibrium, optical and transport tensors, gathered from scientific literature. If a standard magnetic point or space group is defined and a known tensor is selected from the lists the program will obtain the required tensor from an internal database; otherwise, the tensor is calculated live. The working setting is defined by the rules explained [here](#). Live calculation of tensors may take too much time and even exceed the time limit, giving an empty result, if high-rank tensors, a lot of symmetry elements and/or rare settings are introduced.

Additionally, MTENSOR allows the introduction of a space group and a magnetic space subgroup in order to derive the symmetry-adapted form of tensor properties for all the corresponding [domain-related equivalent structures](#).

Further information can be found [here](#)

Introduce a parent space group and a magnetic space subgroup and calculate tensors of the resulting domain-related equivalent structures

Please, enter a magnetic point group by one of these ways:

Choose a magnetic point group:

Magnetic Point or Space Group number: Please, enter the label of the group or choose it

(point group)

Non-conventional setting

Introduce a magnetic point group by hand:

Please, choose a tensor by one of these ways:

MTENSOR

Magnetolectric tensor:

Group 6/m' (#23.4.85)

α^T_{ij}	j			
	1	2	3	
i	1	α^T_{11}	α^T_{12}	0
	2	$-\alpha^T_{12}$	α^T_{11}	0
	3	0	0	α^T_{33}

Number of independent coefficients: 3

Group 622 (#24.1.87)

α^T_{ij}	j			
	1	2	3	
i	1	α^T_{11}	0	0
	2	0	α^T_{11}	0
	3	0	0	α^T_{33}

Number of independent coefficients: 2

Group 62'2' (#24.4.90)

α^T_{ij}	j			
	1	2	3	
i	1	0	α^T_{12}	0
	2	$-\alpha^T_{12}$	0	0
	3	0	0	0

Number of independent coefficients: 1

Group 6mm (#25.1.91)

α^T_{ij}	j			
	1	2	3	
i	1	0	α^T_{12}	0
	2	$-\alpha^T_{12}$	0	0
	3	0	0	0

Number of independent coefficients: 1

Group 6m'm' (#25.4.94)

α^T_{ij}	j			
	1	2	3	
i	1	α^T_{11}	0	0
	2	0	α^T_{11}	0
	3	0	0	α^T_{33}

Number of independent coefficients: 2

Group -6'm'2 (#26.3.97)

α^T_{ij}	j			
	1	2	3	
i	1	α^T_{11}	0	0
	2	0	α^T_{11}	0
	3	0	0	α^T_{33}

Number of independent coefficients: 2

Group -6'm'2' (#26.4.98)

α^T_{ij}	j			
	1	2	3	
i	1	0	α^T_{12}	0
	2	$-\alpha^T_{12}$	0	0
	3	0	0	0

Number of independent coefficients: 1

Group 6/m'mm (#27.3.102)

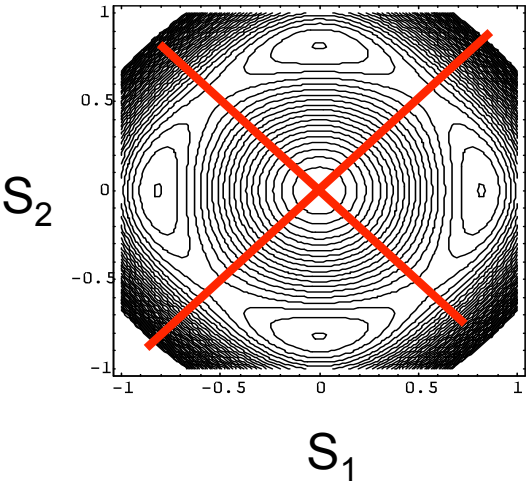
α^T_{ij}	j			
	1	2	3	
i	1	0	α^T_{12}	0
	2	$-\alpha^T_{12}$	0	0
	3	0	0	0

Number of independent coefficients: 1

Ferroic properties

A "multiferroic":
improper ferroelectric

HoMnO₃



param. phase antiferrom. phase

$Pnma\ 1'$ \longrightarrow $P_a n m 2_1$ index 4 \longrightarrow **4 total number of domains**

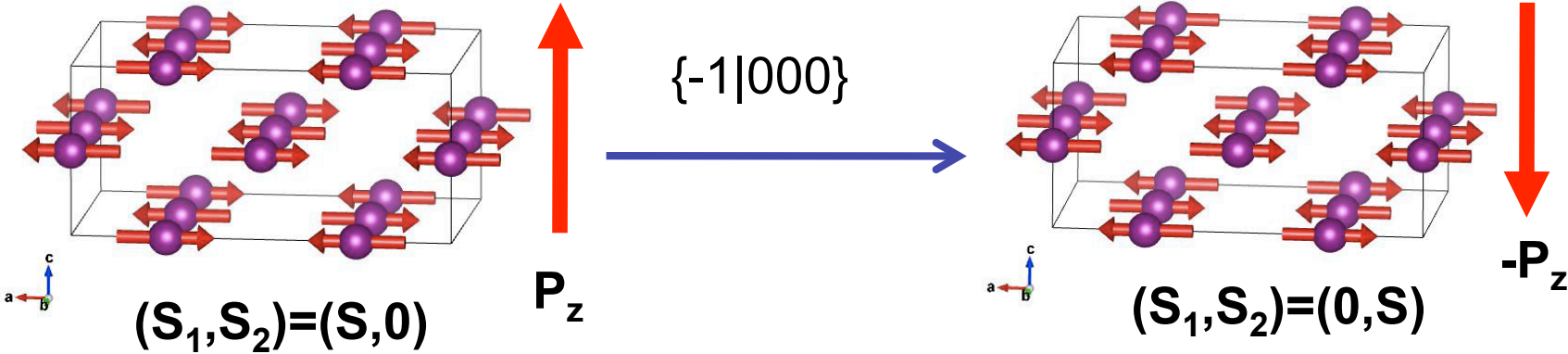
point groups
 $mmm1'$ \longrightarrow $mm21'$ index 2 \longrightarrow **2 ferroic domains**

Secondary symmetry-allowed effect: spontaneous polarization: P_z

$$Pnma1' = P_a n m 2_1 + (1'|000)P_a n m 2_1 + (-1|000)P_a n m 2_1 + (-1'|000)P_a n m 2_1$$

S_1
generators of the four domain configurations:

$$\{g_n\} = \{(1|000), (-1|000), (1'|000), (-1'|000)\}$$



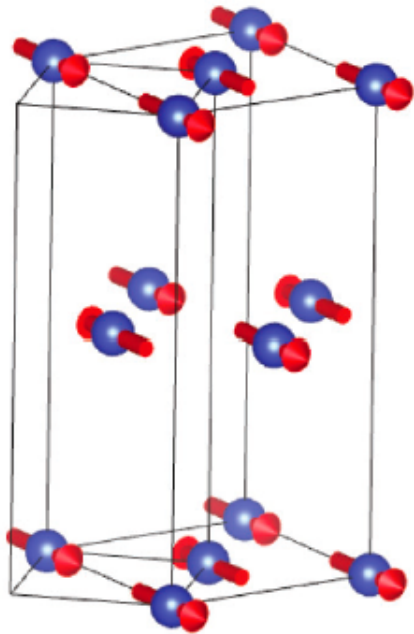
domains: equivalent energy minima

Their MSG are equivalent, but not equal in general

The importance of non-magnetic atoms:

The same spin arrangement can produce different MSGs (and different ferroic properties) depending on the symmetry of the parent structure

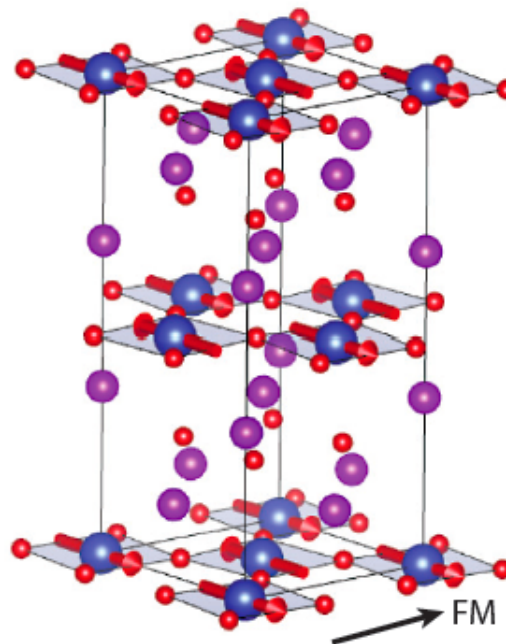
I4/mmm, $k=(1/2, 1/2, 0)$



C_Accm
($c, a - b, a + b; 1/4, 3/4, 1/4$)



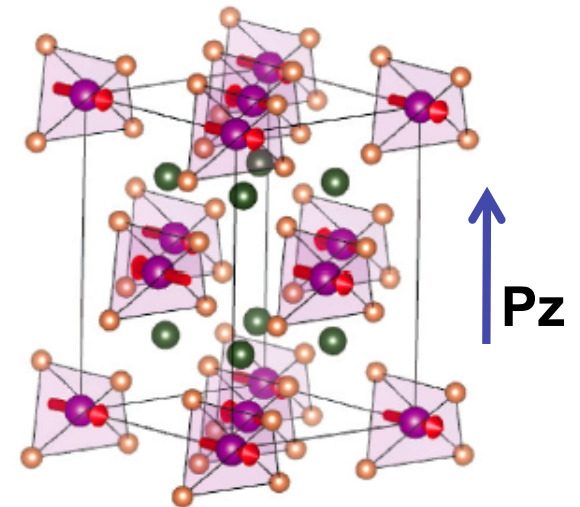
Cmce, $k=(0, 0, 0)$



$Cm'ca'$
($c, a - b, a + b; 0, 0, 0$)



I-42m, $k=(1/2, 1/2, 0)$



A_Bma2
($a + b, -a + b, c; 1/2, 0, 0$)

Hypothetical spin configuration on a structure of type GaMnSe₄

Check the symmetry restrictions on the Mn and Ho spins resulting from the symmetry of the magnetic phase of HoMnO_3 .

4. Upload the mcif file of the magnetic structure of HoMnO_3 in [STRCONVERT](#). Introduce a non-zero spin component of Ho along x or along z, and try to obtain all the atomic positions and moments within the unit cell by transforming to trivial symmetry P1. Check that the program in both cases (for Ho moment along x or z) gives an error/warning: this means that if the Ho atoms are magnetically ordered in this phase, their magnetic moments can only be directed along b. Come back to the description using the MSG using the back button of the browser and introduce now a non-zero spin component for the Ho atoms along b and transform again to P1 to observe the resulting values for the symmetry-related Ho atoms within the unit cell. Visualize the magnetic structure reading the mcif file with VESTA. Using alternatively MVISUALIZE observe the differences between the parent unit cell, the standard unit cell and the unit cell actually used in the description. (file required: *2.HoMnO3.mcif*)

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bilbao crystallographic server

[The crystallographic site at the Condensed Matter Physics Dept. of the University of the Basque Country]

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