Hybrid Simulation: longitudinal particles, transverse envelope

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TRIUMF

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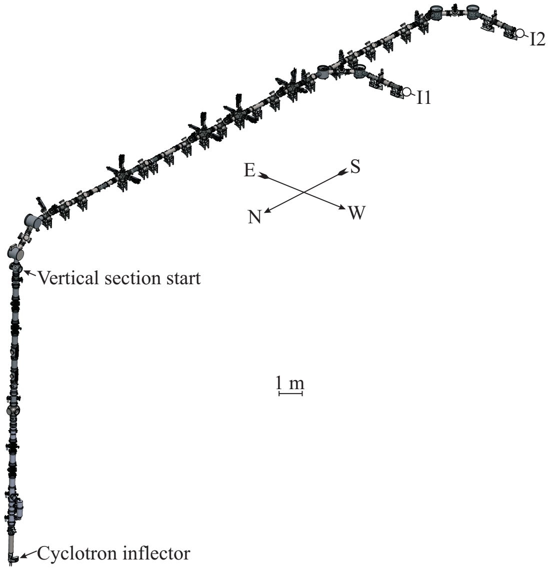
TRIUMF Cyclotron Injection Line



SPUNCH

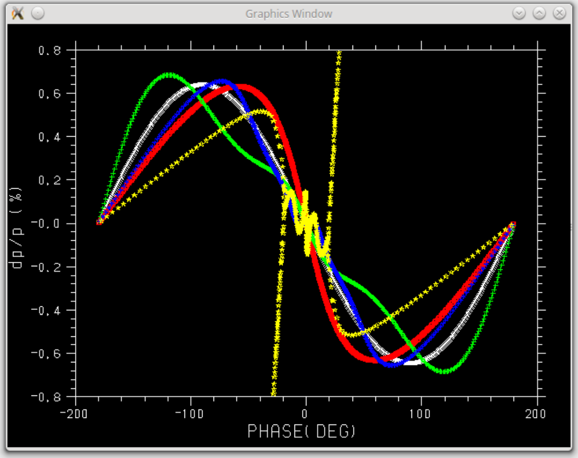


Figure: SPUNCH 1D particle-particle calculation of the injection line.

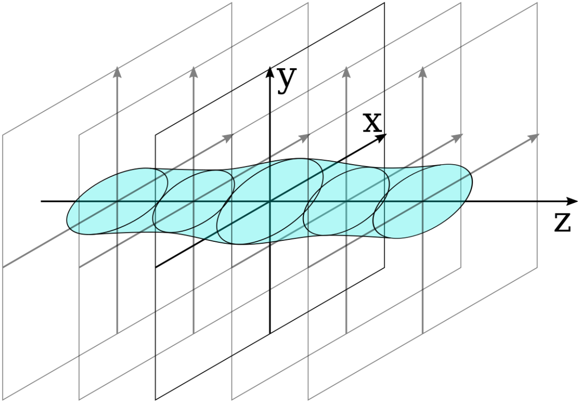
Hybrid Scheme

Ingredients:

- Vlasov–Poisson system
- Non-canonical Hamiltonian formulation
- Completely uncorrelated motion in x and y planes
- Non-relativistic
- Beam is centred on-axis

Hybrid Scheme

1D macro-particles $\otimes$ 2D transverse envelopes.



Hybrid Scheme

$$f(\mathbf{x}, \mathbf{p}; t) = \sum_{i=1}^{N_\mathsf{p}} w_i f_i^{\perp}(x, y, p_x, p_y; t) R(z - z_i) \delta(p - p_i)$$

$$f_i^{\perp}(x, y, p_x, p_y; t) \sim \{\langle x^2 \rangle_i, \langle x p_x \rangle_i, \langle p_x^2 \rangle_i, \langle y^2 \rangle_i, \langle y p_y \rangle_i, \langle p_y^2 \rangle_i\}$$

Hybrid Scheme

$$\mathbf{Q}_i = \begin{bmatrix} \sqrt{\langle x^2 \rangle_i} \\ \sqrt{\langle y^2 \rangle_i} \\ \langle z \rangle_i \end{bmatrix} \qquad \mathbf{P}_i = \begin{bmatrix} w_i \frac{\langle x p_x \rangle_i}{\sqrt{\langle x^2 \rangle_i}} \\ w_i \frac{\langle y p_y \rangle_i}{\sqrt{\langle y^2 \rangle_i}} \\ w_i \langle p_z \rangle_i \end{bmatrix}$$

$$\boldsymbol{\mathcal{E}}_i = \begin{bmatrix} \sqrt{\langle x^2 \rangle_i \langle p_x^2 \rangle_i - \langle x p_x \rangle_i} \\ \sqrt{\langle y^2 \rangle_i \langle p_y^2 \rangle_i - \langle y p_y \rangle_i} \end{bmatrix}$$

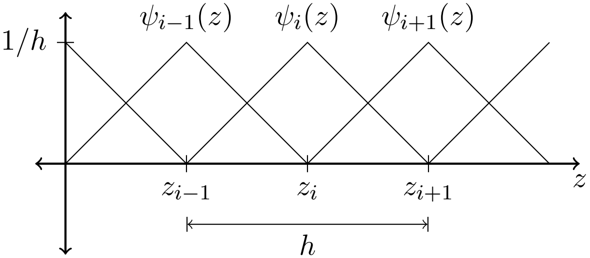
Hybrid Scheme

$$\hat{H} = \sum_{i=1}^{N_{\mathsf{p}}} \left[\frac{w_i}{2m} \left((P_i^x)^2 + (P_i^y)^2 + (P_i^z)^2 + \left[\frac{\mathcal{E}_i^x}{Q_i^x} \right]^2 + \left[\frac{\mathcal{E}_i^y}{Q_i^y} \right]^2 \right) \right.$$
$$\left. + q w_i \int dz\, R(z - Q_i^z) \left(\phi + (Q_i^x)^2 \, \partial_{xx} \phi + (Q_i^y)^2 \, \partial_{yy} \phi \right) \right]$$

Note: $\partial_{xx} = \frac{\partial^2}{\partial x^2}$

Electrostatic Potential Solver

$$\phi(\mathbf{x}, t) = \sum_{i=1}^{N_g} \varphi_i(\mathbf{x}^{\perp}, t)\psi_i(z)$$



Electrostatic Potential Solver

$$\phi(\mathbf{x}) = \sum_{i,j,k=1}^{N_g} S_{ij} S_{jk}\, \psi_i(z)\, \left(G_j * \rho_k\right)(\mathbf{x}^\perp)$$

$$\begin{bmatrix} \phi \\ \partial_{xx}\phi \\ \partial_{yy}\phi \end{bmatrix} = \sum_{i,j,k=1}^{N_g} \underbrace{S_{ij} S_{jk}}_{\mathcal{O}(N_{\mathsf{g}}^3)}\, \psi_i(z)\, \underbrace{\begin{bmatrix} (G_j * \rho_k) \\ \partial_{xx}\,(G_j * \rho_k) \\ \partial_{yy}\,(G_j * \rho_k) \end{bmatrix}_{(\mathbf{x}^\perp = \mathbf{0})}}_{\mathcal{O}(N_{\mathsf{g}}^2)}$$

FODO Test

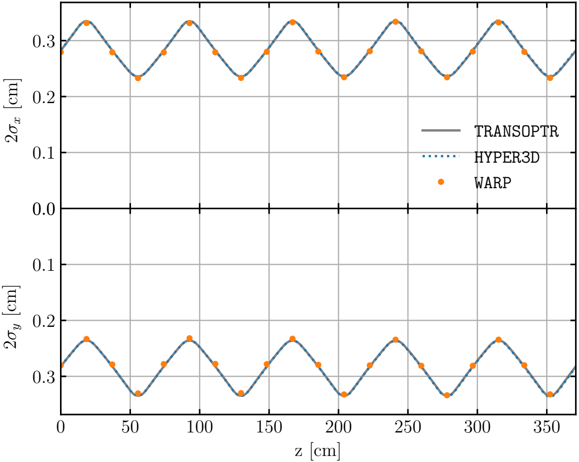


Figure: FODO test tuned to agree with TRANSOPTR. WARP needs $N_\text{p} = 10^6$, HYPER3D needs $N_\text{p} = 50$. Both use $N_\text{g} = 64$.

Performance Comparison

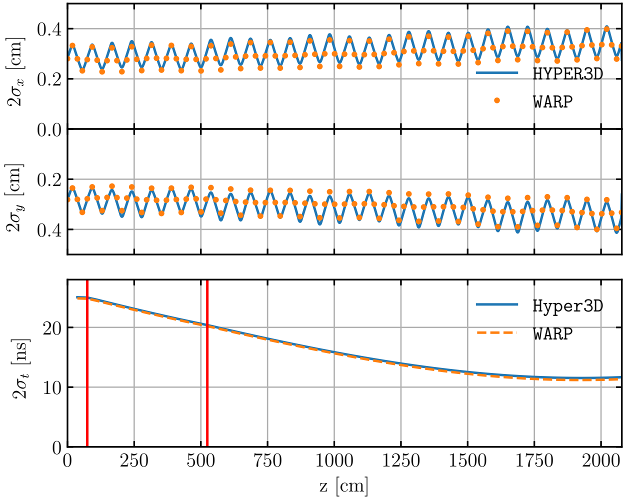
FODO Simulation

WARP: $N_{\mathsf{p}} = 10^5$; $N_{\mathsf{g}}^{(z)} = 64$;

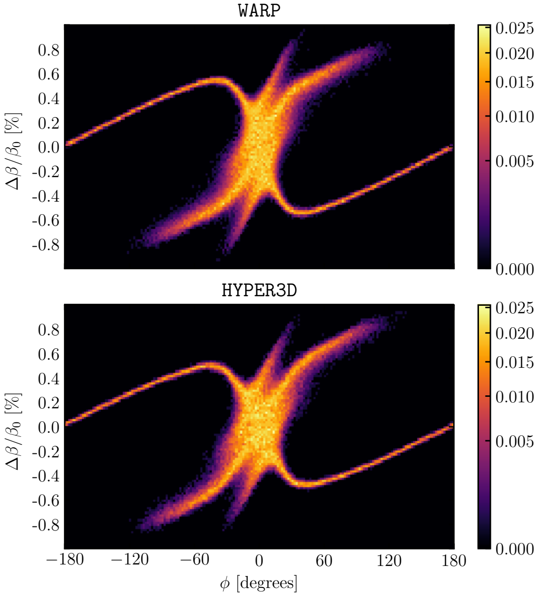
HYPER3D: $N_{\mathsf{p}} = 50$; $N_{\mathsf{g}}^{(z)} = 64$;

	WARP	HYPER3D
Avg. Time Step	19.8 ms	1.68 ms

Injection Line: FODO + Bunchers



Injection Line: FODO + Bunchers



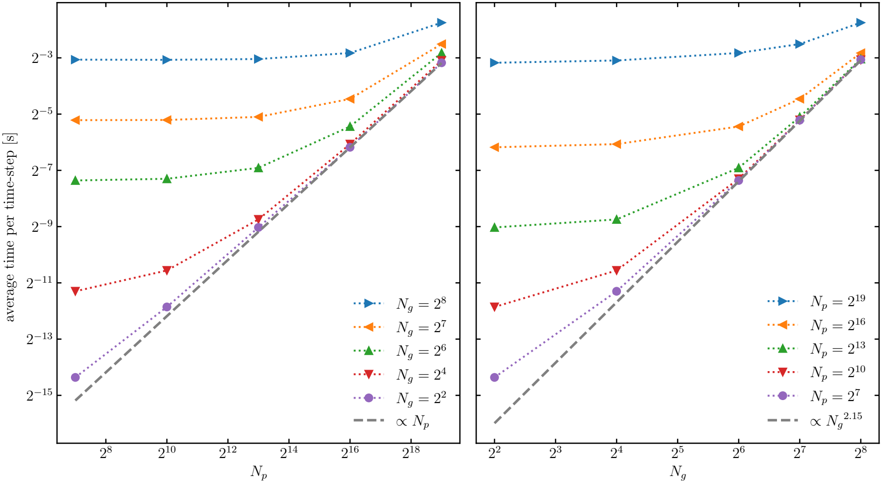
Performance Comparison

Injection Line Simulation

Multithreaded: 4 cores; $N_{\mathsf{g}}^{(z)} = 128$; (WARP: $N_{\mathsf{g}}^{(x,y)} = 32$)

	WARP	HYPER3D	HYPER3D
N_{p}	10^5	10^5	$5 \cdot 10^3$
Total Runtime	94.4 s	79.0 s	15.1 s
Avg. Time Step	50.8 ms	43.5 ms	9.7 ms

HYPER3D serial performance



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Hybrid macroparticle algorithm for modeling space charge

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A space charge algorithm is constructed that is a hybrid between envelope and multiparticle models. The transverse dynamics is simplified by tracking the transverse envelope of each macroparticle. The equations of motion are derived from the Hamiltonian-fluid formulation of the Vlasov Poisson system. A novel electrostatic self-field solver is derived that solves directly for the self-consistent on-axis potential and linear defocusing force, including longitudinal and beam pipe boundary conditions. The implementation of the algorithm, HYPER3D, is presented. It is tested against the particle-in-cell code WARP, using various configurations of a periodic focusing structure with rf cavities. The required number of macroparticles is reduced substantially compared to standard particle-tracking codes. This model is well adapted to cases where the transverse dynamics is linear and where the details of the longitudinal dynamics are the principal point of interest; an example is where a dc beam is converted to a bunched beam.

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I. INTRODUCTION

Many reduced dimension models exist to study space-

yielding performance advantages. The reduced model, SPUNCH [5], was designed to study longitudinal dynamics. It tracks uniform discs of charge, with a fixed radius

Link: Phys Rev. A.B.

Bridging the gap in space charge dynamics

To redesign the H^- ion injection line of the TRIUMF cyclotron a new code was produced that uses second moments to simplify the transverse dynamics.

There is a gap between the speed of the envelope method and the precision of macro-particle codes. There is potential for these moment-reduced methods to provide only the necessary precision.

Thank You

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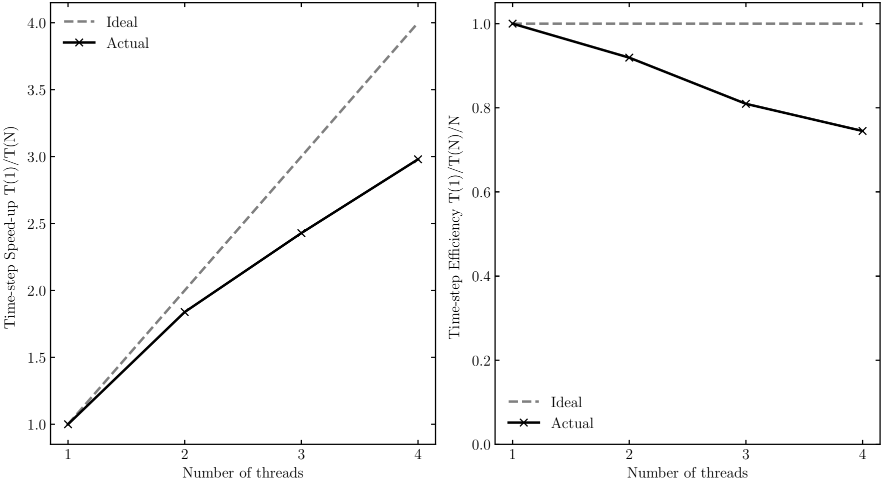
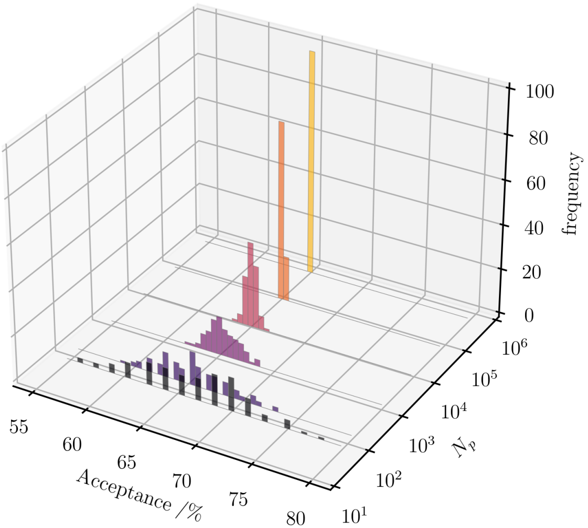
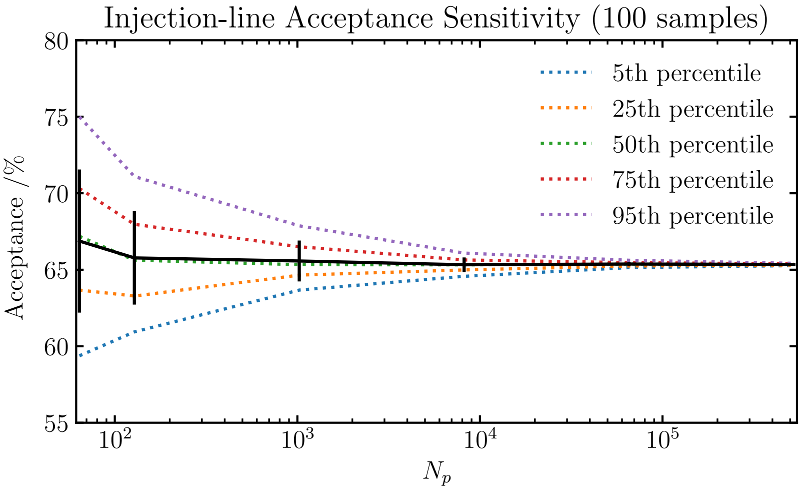
HYPER3D parallel performance



The lower limit on N_p



The lower limit on N_p



Symplecticity?

$$H[f] = \frac{1}{2} \int d^3\mathbf{x}'\, d^3\mathbf{p}'\, d^3\mathbf{x}''\, d^3\mathbf{p}'' \left[f(\mathbf{x}', \mathbf{p}') G(\mathbf{x}', \mathbf{x}'') f(\mathbf{x}'', \mathbf{p}'') \right]$$

$$\Longrightarrow \frac{\delta H}{\delta f(\mathbf{x}, \mathbf{p})} = \frac{1}{2} \int d^3\mathbf{x}''\, d^3\mathbf{p}'' \left[G(\mathbf{x}, \mathbf{x}'') f(\mathbf{x}'', \mathbf{p}'') \right] + \frac{1}{2} \int d^3\mathbf{x}'\, d^3\mathbf{p}' \left[f(\mathbf{x}', \mathbf{p}') G(\mathbf{x}', \mathbf{x}) \right]$$

If G is symmetric:

$$\frac{\delta H}{\delta f(\mathbf{x}, \mathbf{p})} = \int d^3\mathbf{x}' \left[G(\mathbf{x}, \mathbf{x}') \rho(\mathbf{x}') \right] = \phi(\mathbf{x})$$

Model Complexity

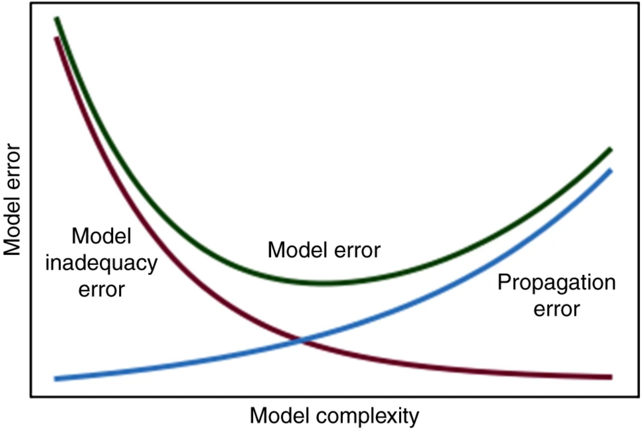


Figure: Figure from A. Saltelli, *A short comment on statistical versus mathematical modelling*, Nature communications 2019.