Nested Airbag Model for Study of Transverse Coherent Instabilities

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U.S. DEPARTMENT OF ENERGY | Office of Science

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Summary slide, 5th ICFA mini-workshop on Space Charge
Theme: Bridging the gap in space charge dynamics

In 1-2 sentences, summarize the content of this presentation
(If relevant, specify type of facility, species, tune shift):

The Multi Loop Square Well Model (MLSW) is a semi-analytic model for studying mode coupling instabilities in synchrotrons. This makes it possible to better understand instabilities at large tune shifts, wakes and arbitrary chromaticity.

From your perspective, where is the gap regarding space charge effects?
(understanding/control/mitigation/prediction/?)

We need to better understand the range of applicability for approximations and calculations.

What is needed to bridge this gap?

Careful experiments and full physics simulations are necessary to bridge this gap.

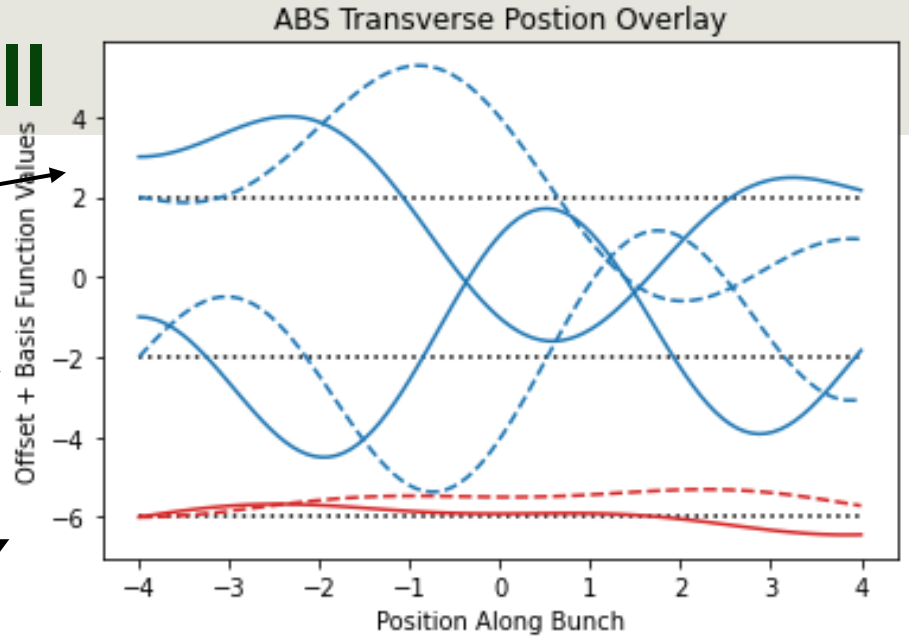
Historical Models for SC TMCI: Square Well Model / Airbag Square Well

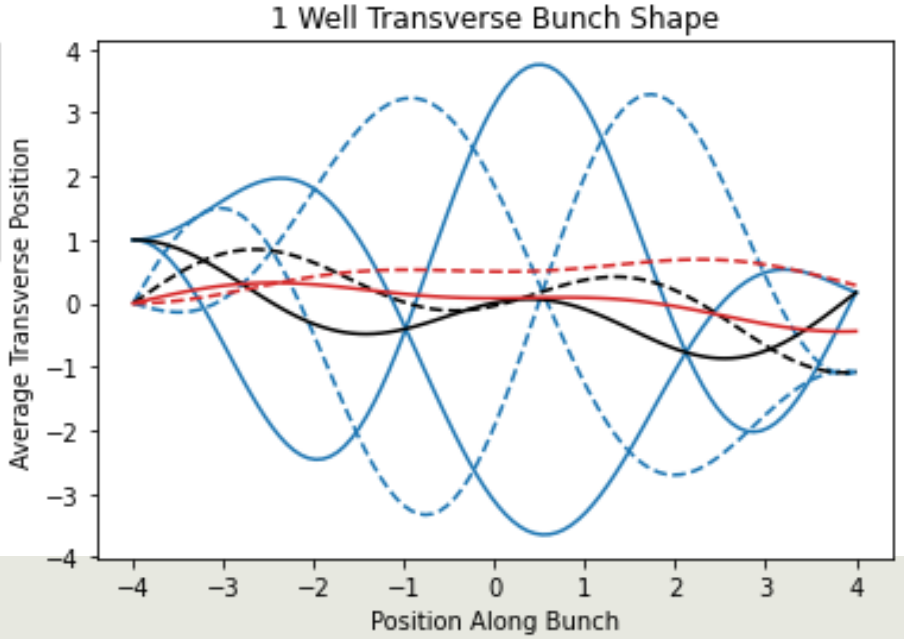
$$\frac{dx_+(\tau)}{d\tau} = -i\left\{\frac{\Delta Q_{sc}/2 + \Delta Q_x - \xi v_0}{v_0}\right\}x_+ - iF/v_0 + i(\Delta Q_{sc}/2v_0)x_-, \quad (34)$$

$$\frac{dx_-(\tau)}{d\tau} = +i\left\{\frac{\Delta Q_{sc}/2 + \Delta Q_x + \xi v_0}{v_0}\right\}x_- + iF/v_0 - i(\Delta Q_{sc}/2v_0)x_+, \quad (35)$$

$$\frac{dF(\tau)}{d\tau} = -\alpha F + (\kappa W_0/2)\{x_- + x_+\}. \quad (36)$$

PRSTAB 1, 044201

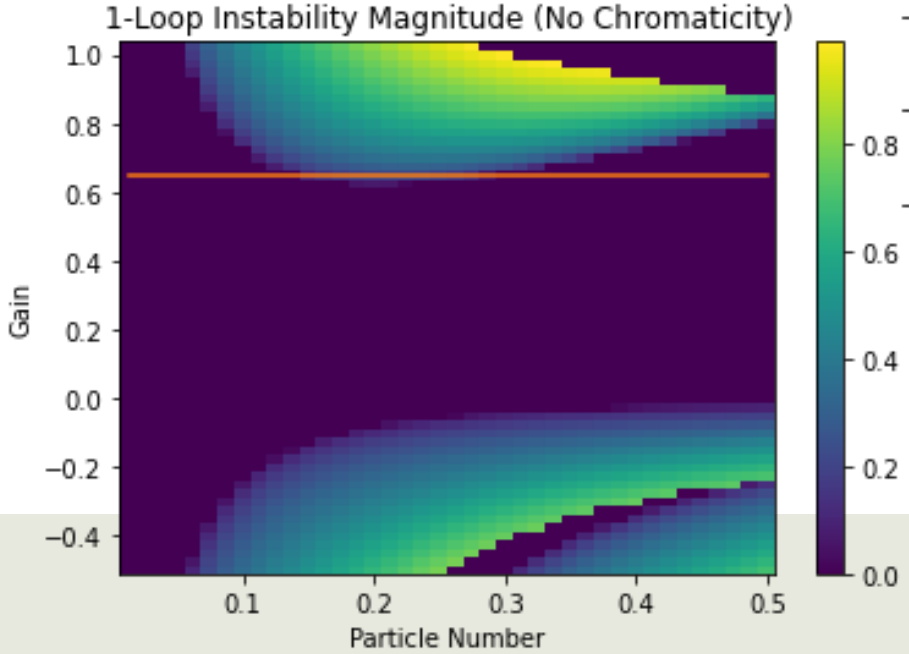




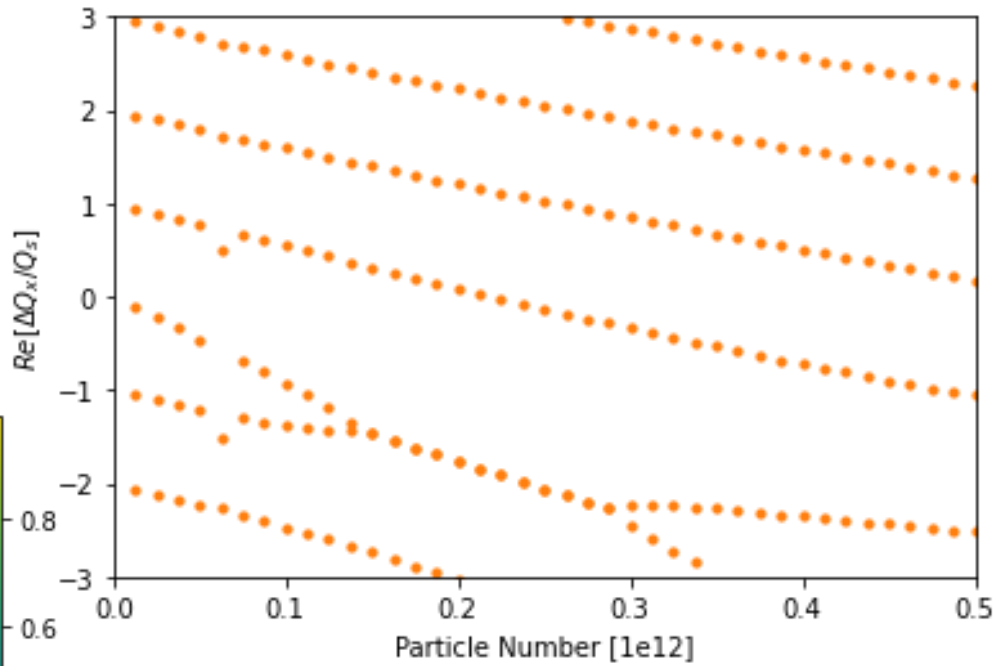
- SWM developed by M. Blaskiewicz as one of few analytic models with long. coupling and trans. coherent SC
- Recent work and analysis using this method from Burov under the name Airbag Square Well (ABS).

SWM/ABS Instability Thresholds

- SWM solved in terms of parameter ΔQ_x, the tune shift of the system
- Each solution is an independent mode which satisfies the boundary conditions
- Collective motion of the beam sum of modes
- Modes couple into one another driving imaginary (unstable) tune components

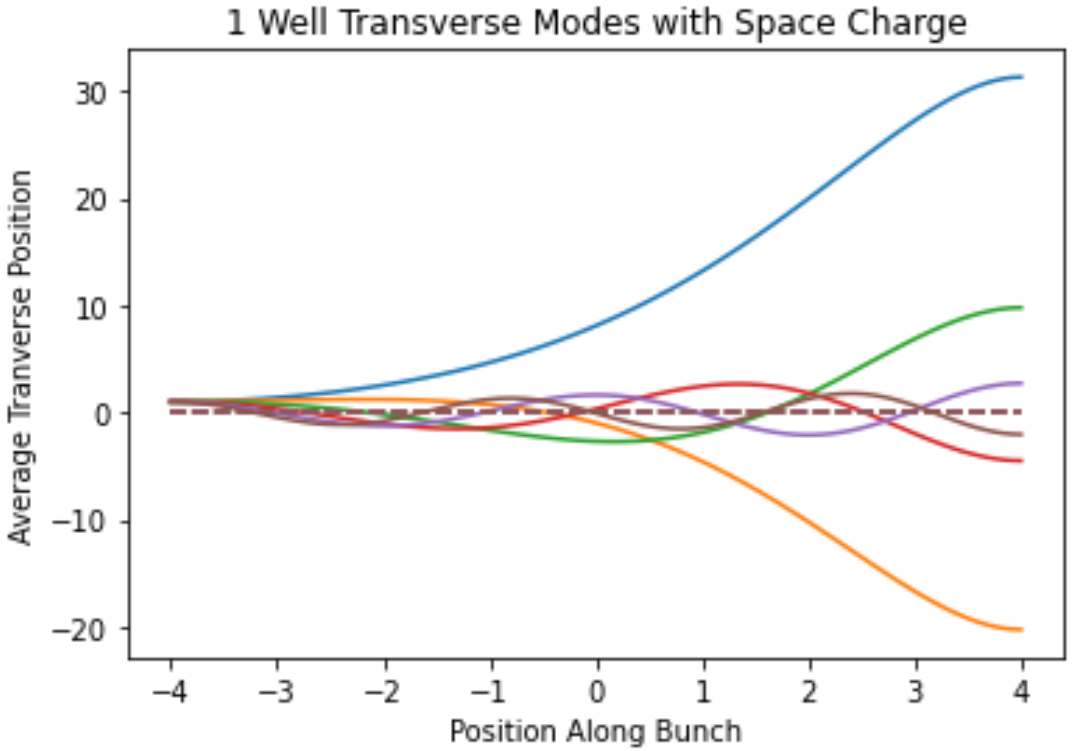


ABS Instability Diagram

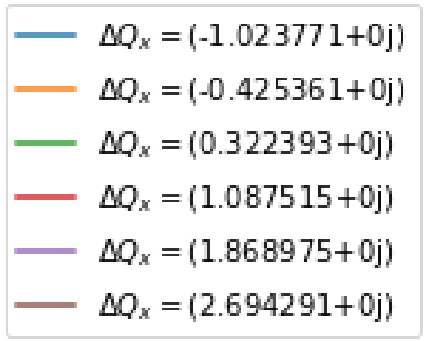


ABS at Strong Space Charge: Convective Instability

- Bunches can be spatially unstable, with large head tail amplification
- Head tail amplification generally scales with ΔQ_{sc}



Each color is a valid ΔQ_x solution



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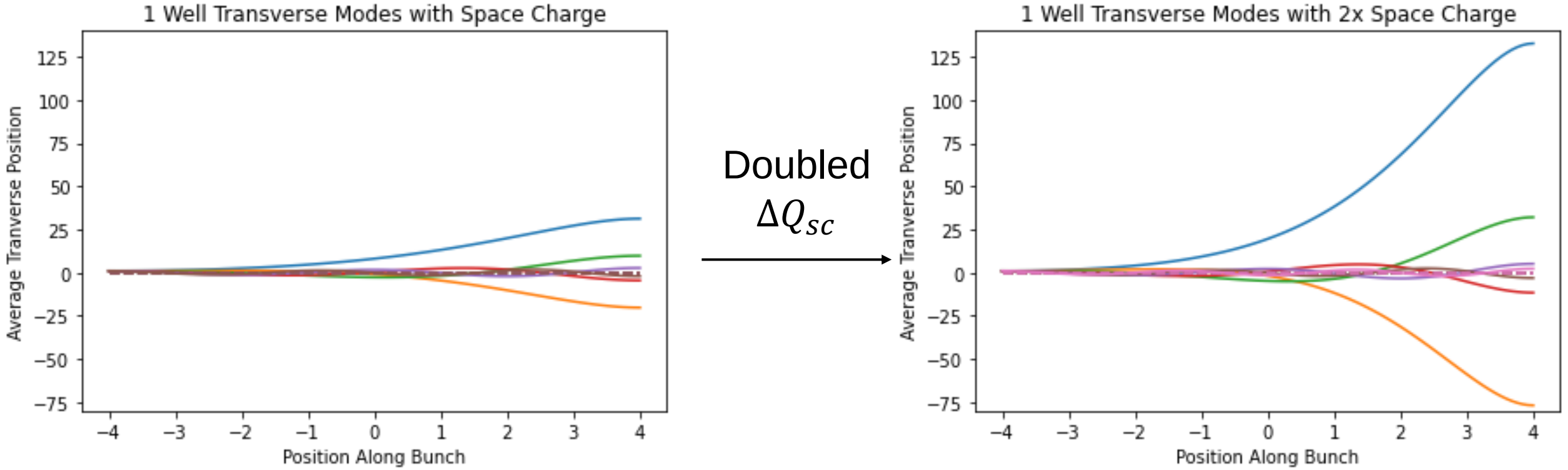
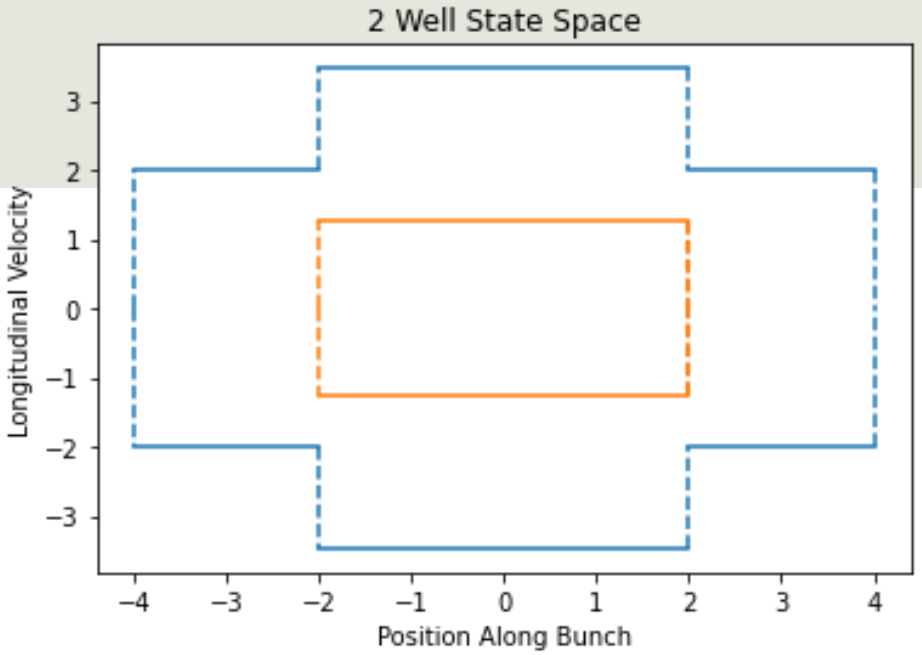
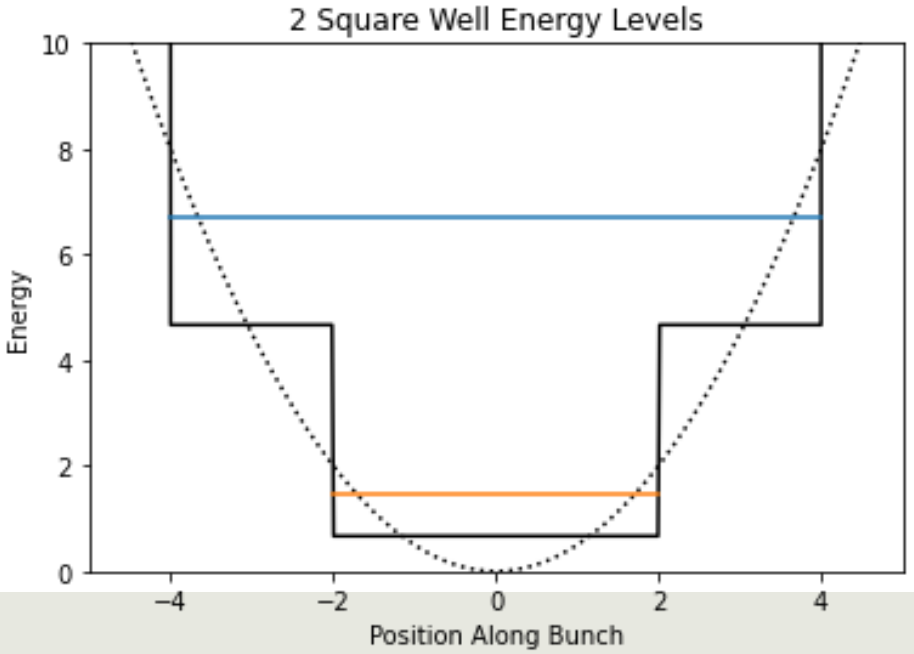


Extending Longitudinally: Multi Loop Square Well

- ABS/SWM can be thought of as a limiting case of a peaked bunch distribution.

- Nested square wells with multiple loops of particles create more realistic phase space distribution





Collective EOMs with SC

Average Transverse Position eq.

$$\frac{d\hat{X}_j^n}{dz} = \frac{i\omega_0}{\dot{z}_j^n}[h_1(z,\dot{z}_j^n)\hat{X}_j^n + h_2(t,z)] \quad (33)$$

$$h_1(z,\dot{z}_j^n) = \xi_1 \dot{z}_j^n + \Delta Q'_{sc} \sum_{j=1}^{m} \lambda_j^n + \Delta Q_x \quad (34)$$

$$h_2(z) = -\Delta Q'_{sc} \sum_{j=1}^{m} \lambda_j^n (\hat{X}_j^n + \hat{X}_{-j}^n) + \frac{1}{2Q_x} \sum_{k=1}^{\kappa} \hat{F}_k^n \quad (35)$$

Wake Force eq.

$$\frac{d\hat{F}_k^n}{dz} = w_k \sum_{j=1}^{m} \lambda_j^n (\hat{X}_j^n + \hat{X}_{-j}^n) - \alpha_k \hat{F}_k^n \quad (36)$$

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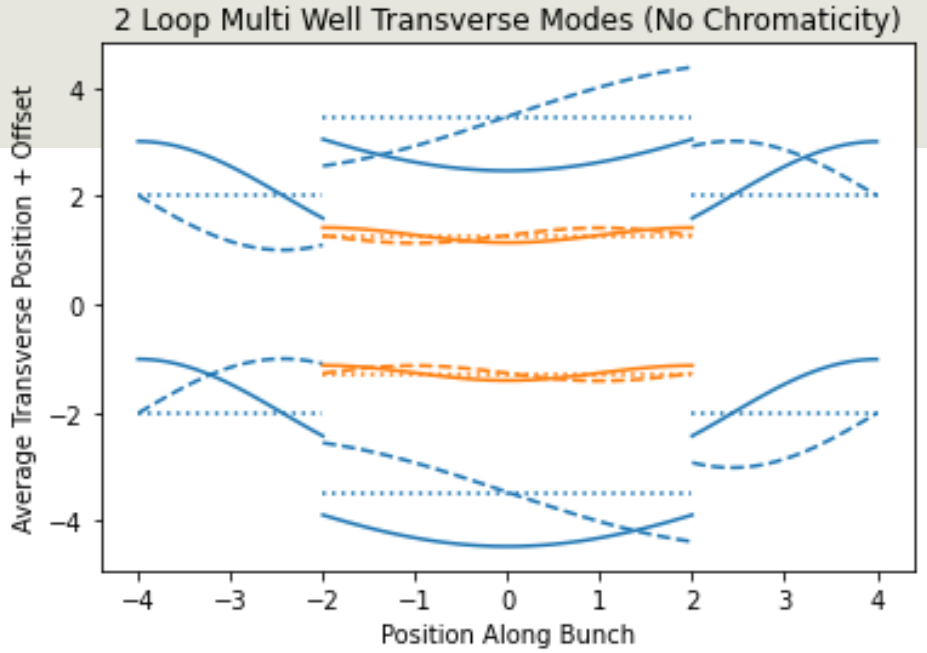

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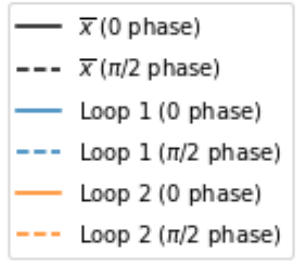
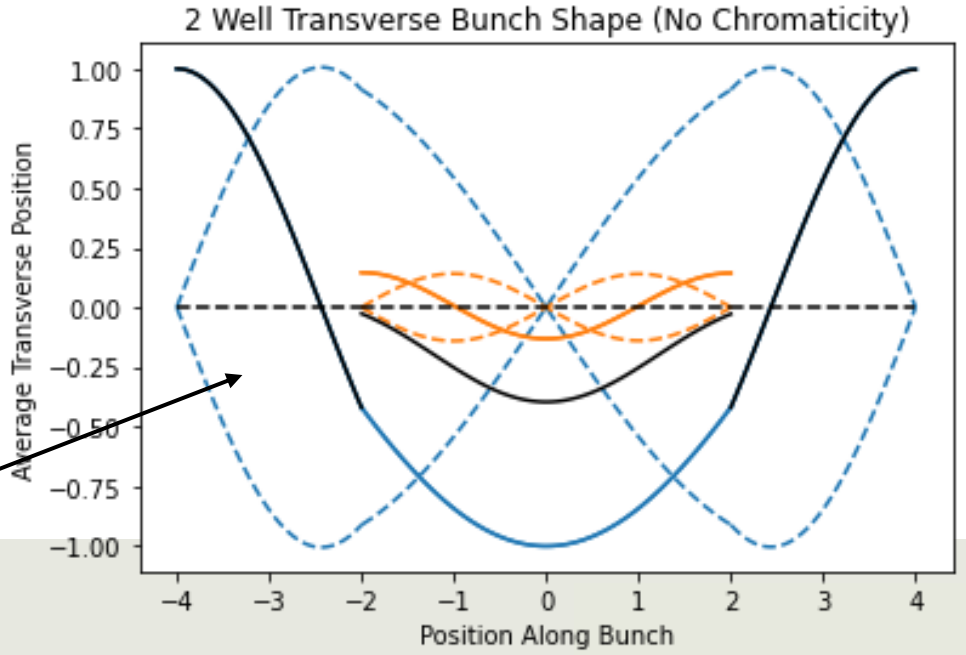
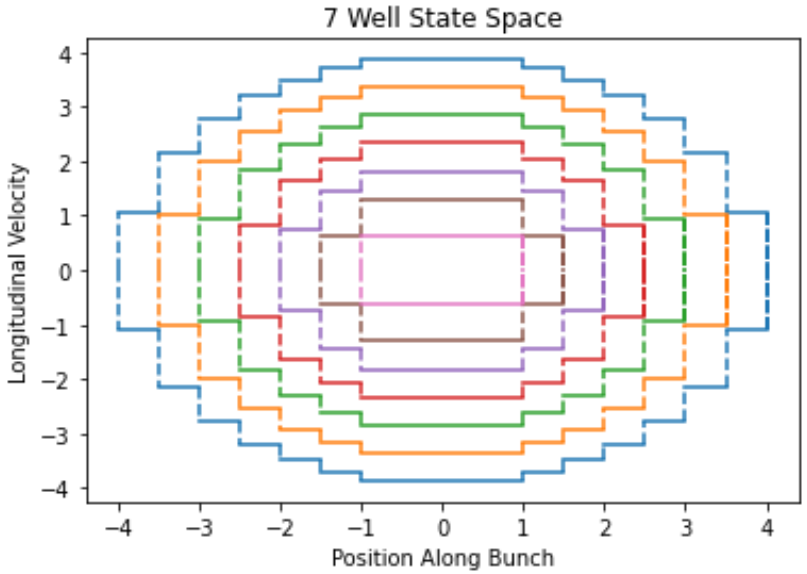


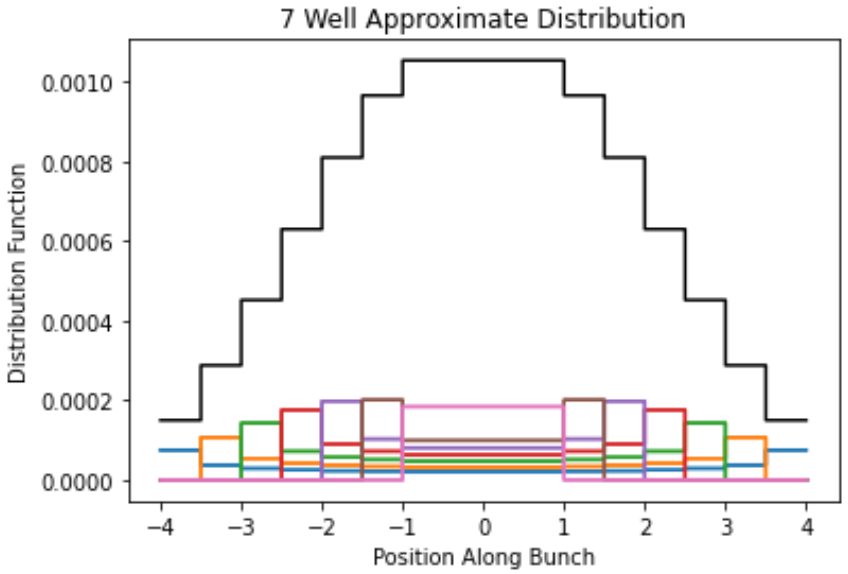


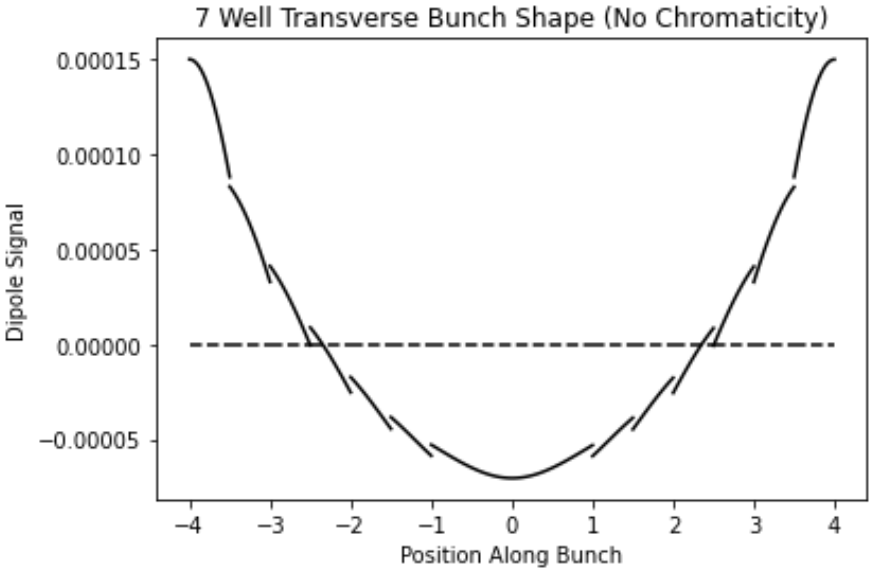
Jump due to discretization

Increasing Well Number

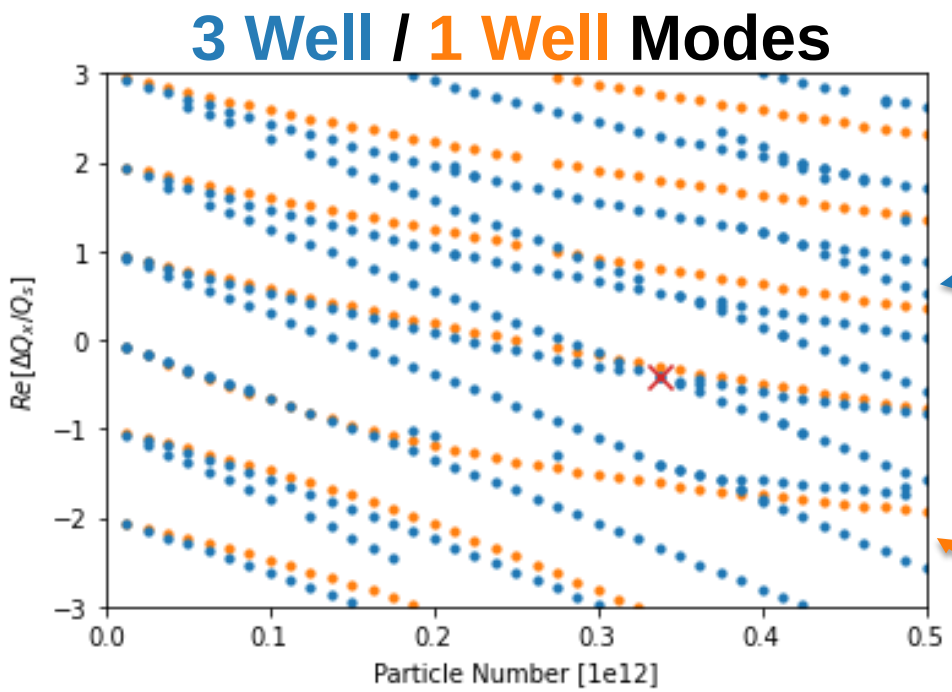
- The number of potential wells and loops of current can be arbitrarily high—only limited by computation time
- Converges with sufficient wells
- Beam bunches can have range of synchrotron tunes, not possible SWM/ABS

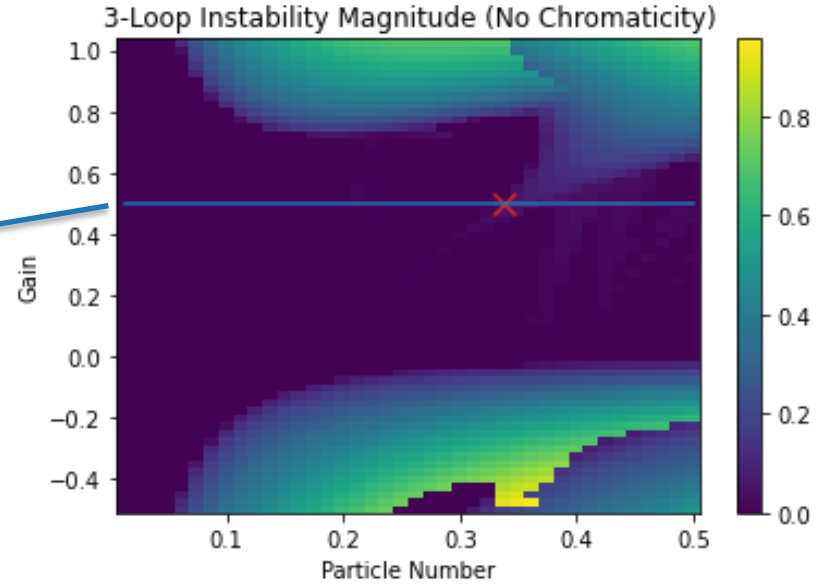


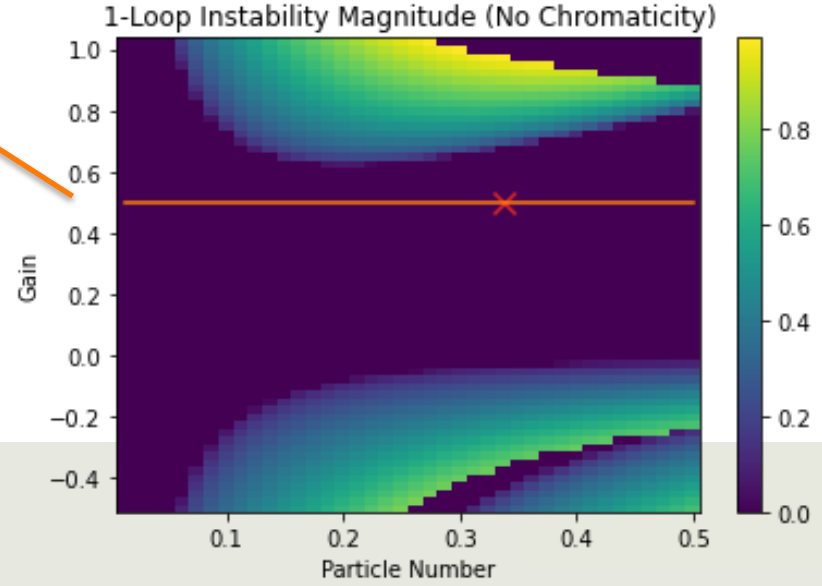




3 Multi / 1 Well Comparison







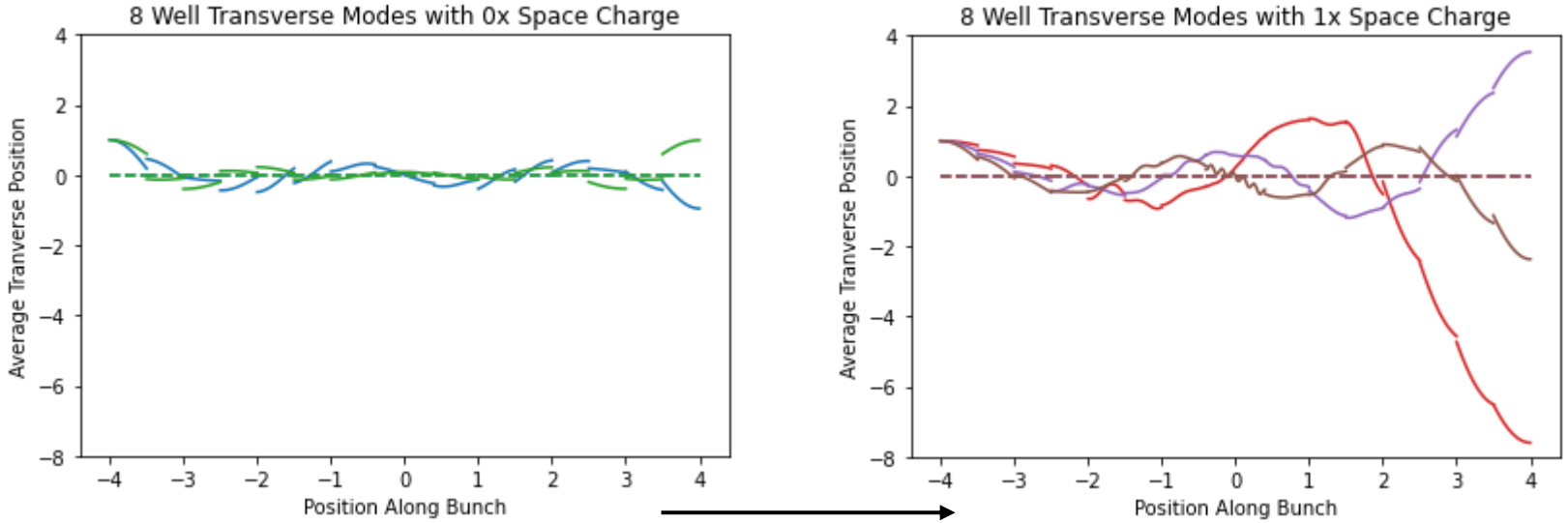
- Multiwells have additional sideband modes
- New sidebands create new avenues for absolute instabilities

8 Potential Well Convective Modes

Each color is a valid ΔQ_x solution

- Previously unclear how the convective instability presents with more complex longitudinal physics
- New Absolutely near convective behavior makes observation difficult
- Convective modes still exist, but much more difficult to parse

8 Potential Well Convective Modes



ΔQ_{sc}
Turned On

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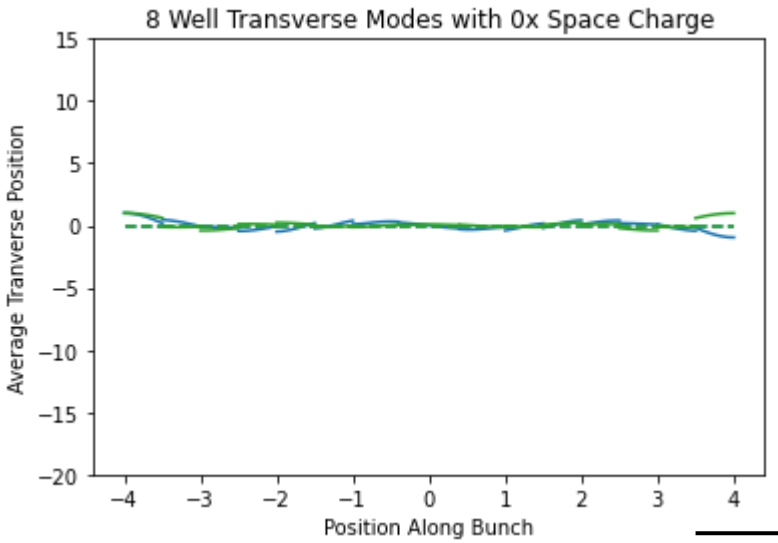

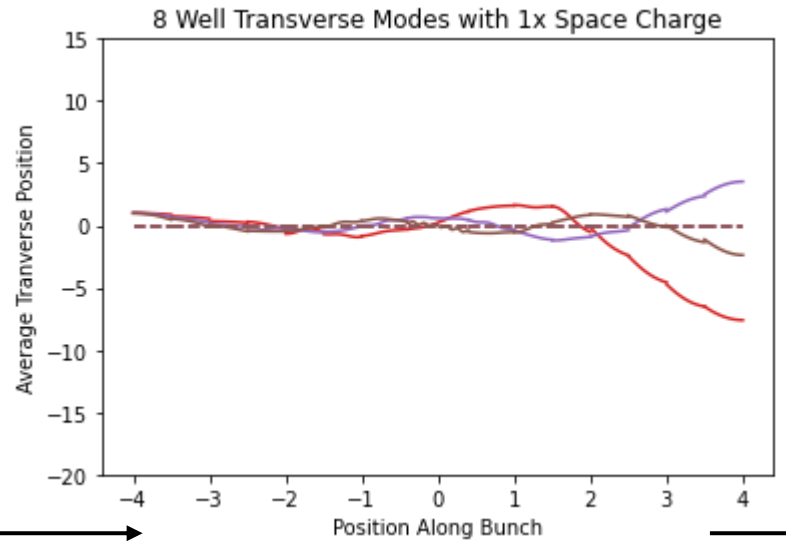

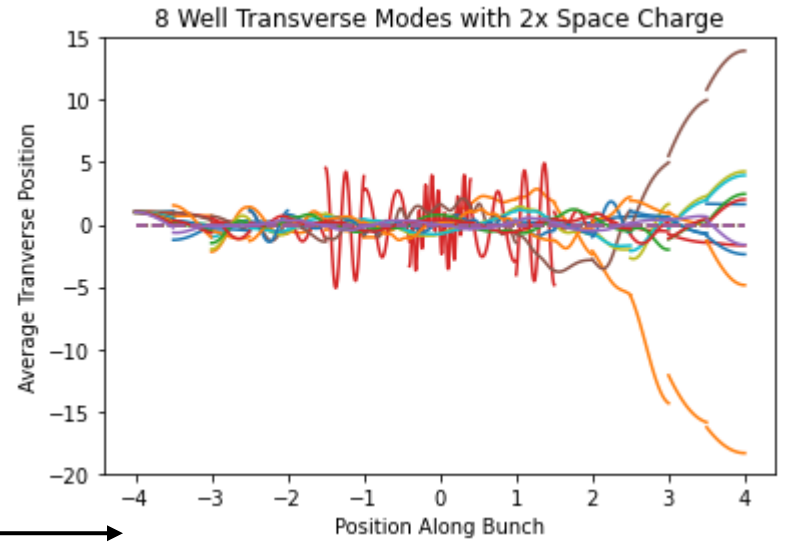

ΔQ_{sc} Turned On

ΔQ_{sc} Doubled

- Previously unclear how the convective instability presents with more complex longitudinal physics
- New Absolutely near convective behavior makes observation difficult
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Thank You!

Any Questions?

- Work partially supported by the US Department of Energy, Office of Science, High Energy Physics under Cooperative Agreement award number DE-SC0018362 and Michigan State University

Nested Airbag Model for Study of Transverse Coherent Instabilities

For transverse coherent instabilities, longitudinal coupling can have a significant effect on the dynamics. The shape of the beam varies the space charge detuning of particles as they travel along the synchrotron period. Additionally, chromatically induced head tail phase shifts can add instabilities that must be accounted for*. Therefore, models for studying transverse instabilities must be amended and generalized to account for the longitudinal physics of the system. To this end, a generalization of the Square Well Model** (SWM) is described to account for more complex longitudinal dynamics. This generalization adds a series of square potential wells and trapped airbags to approximate an arbitrary longitudinal distribution. Future outlooks and applications of such a model are also discussed.

*PRAB 24, 024402
**PRSTAB 1, 044201