Optimization via adjoint methods of flat-to-round transformers for electron cooling

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Adjoint Approach: What it does

- Finds the dependence of system performance on parameters

- Useful for:

    Optimization – gradient descent
    Sensitivity Studies

- Requires identification of a Figure of Merit (FoM)

$F(\mathbf{a})$ where $\mathbf{a}$ Vector of parameters

- Efficient calculation of $dF(\mathbf{a})/d\mathbf{a}$

Direct evaluation w/N parameters, requires N solutions
Adjoint approach – One or two solutions

How Does It Work?

$\mathbf{X}$ - State of System $\mathbf{a}$ - Vector of Parameters

$\mathbf{A}(\mathbf{X}) = \mathbf{a}$ Nonlinear vector equation

Figure of Merit (FoM) $F(\mathbf{X}, \mathbf{a})$

Linearize: $\mathbf{a} \rightarrow \mathbf{a} + \delta\mathbf{a}$ $\mathbf{X} \rightarrow \mathbf{X} + \delta\mathbf{X}$ $F \rightarrow F + \delta F$

$$\delta F = \delta\mathbf{a} \cdot \frac{\partial F}{\partial \mathbf{a}} + \delta\mathbf{X} \cdot \frac{\partial F}{\partial \mathbf{X}}$$

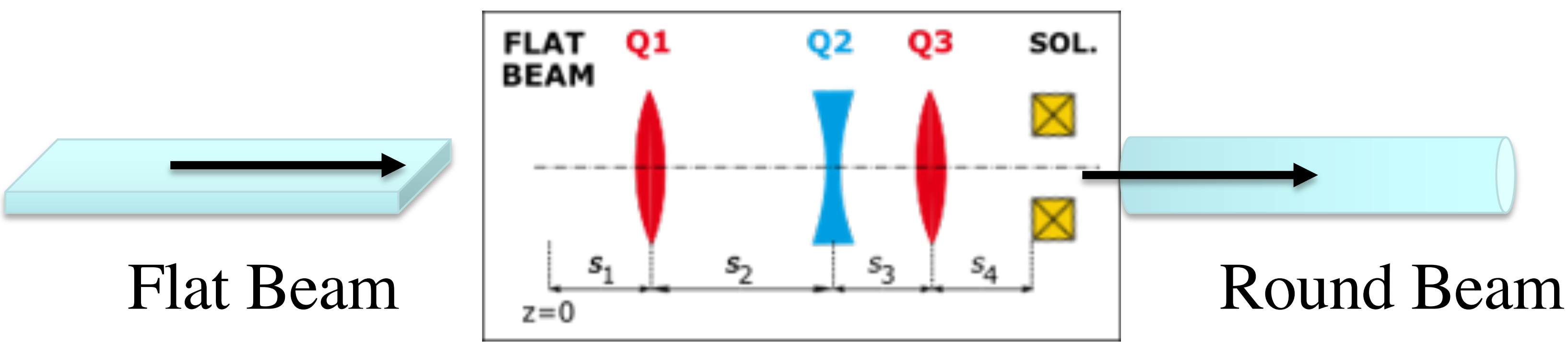
$$\delta\mathbf{X} \cdot \frac{\partial \mathbf{A}}{\partial \mathbf{X}}(\mathbf{X}) = \delta\mathbf{a}$$

Solve Adjoint $\frac{\partial \mathbf{A}}{\partial \mathbf{X}} \cdot \mathbf{Y} = \frac{\partial F}{\partial \mathbf{X}}$

$$\boxed{\delta F = \delta\mathbf{a} \cdot \left[\frac{\partial F}{\partial \mathbf{a}} + \mathbf{Y} \right]}$$

Optimization of Flat to Round Transformers Using Adjoint Techniques*

L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and T. M. Antonsen Jr , Phys Rev Accel and Beams V25, 044002 (2022).



Flat to Round and Round to Flat transformers were proposed for cooling of hadron beams. Ya. Derbenev, Adapting optics for high-energy electron cooling, Tech. Rep. UM-HE-98-04-A (1998).

Steps

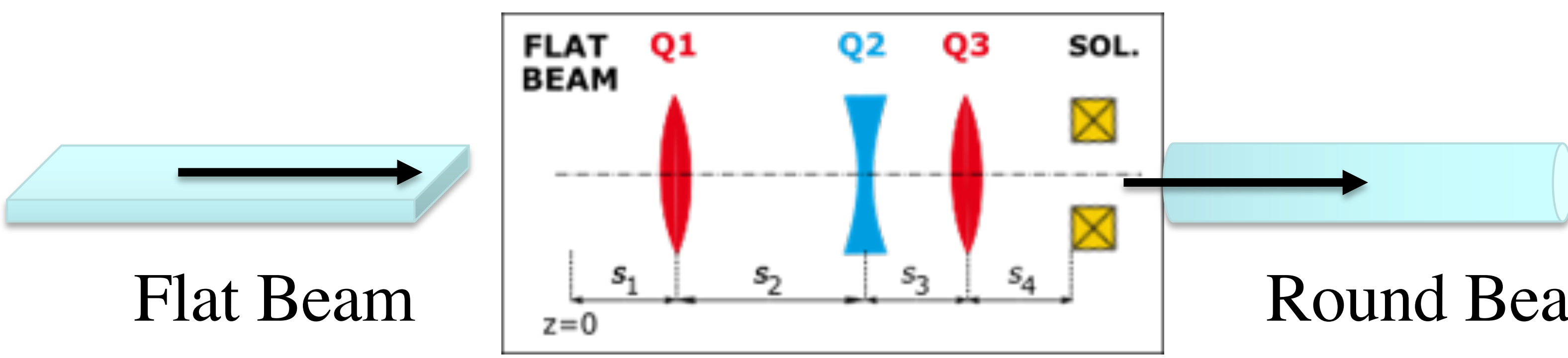
1. Derive system of moment equations (include self fields)

2. Linearize (to compute parameter gradient)

3. Find adjoint system

4. Decide on Figures of Merit

5. Optimize by Gradient Descent



Moment Equations

The second order moments are averages of the 4 by 4 elements of the Sigma matrix.

10 independent moments

$$\underline{\underline{\Sigma}} = \left[\left\langle \begin{matrix} xx & xx' & xy & xy' \\ x'x & x'x' & x'y & x'y' \\ yx & yx' & yy & yy' \\ y'x & y'x' & y'y & y'y' \end{matrix} \right\rangle \right] \quad x' = \frac{dx}{dz}$$

Moments: *$X(z)$*, 10 components

Matrices

$$\frac{d}{dz}\mathbf{X} = \dot{\mathbf{X}}(\mathbf{X}, z | \mathbf{a}) = \mathbf{O}_{magnets}(z | \mathbf{a}) \cdot \mathbf{X}(z) + \mathbf{O}_{SpCh}(\mathbf{X}, z) \cdot \mathbf{X}(z)$$

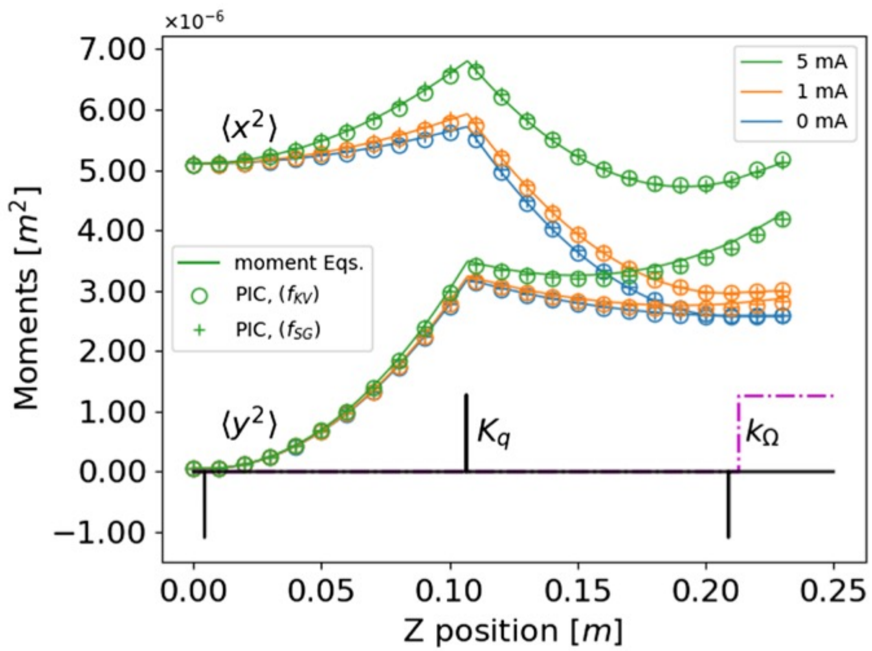
Magnet parameters - **a**

Nonlinear
Assumes K-V

Figure of Merit (FoM): $F(\boldsymbol{X}(z_f))$

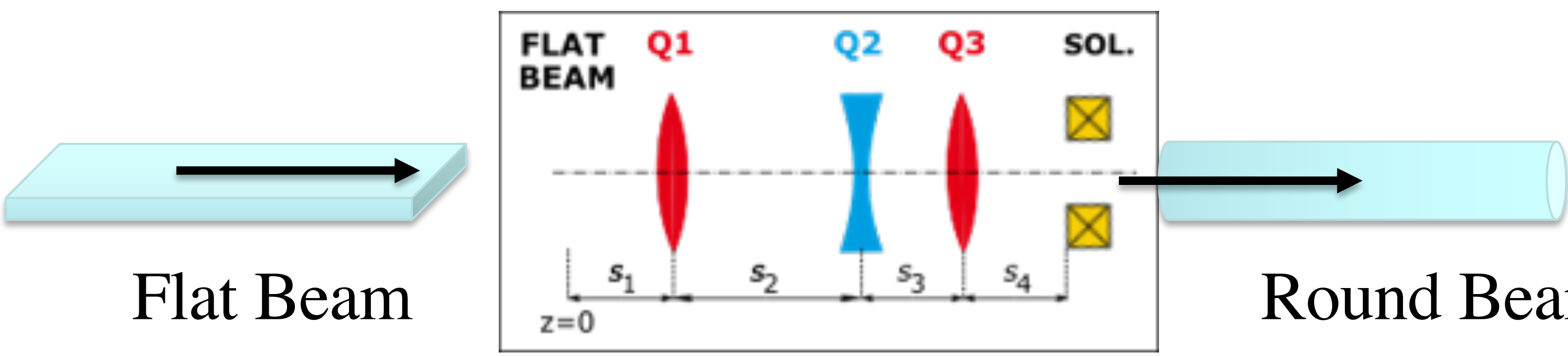
Function of moments at plane $z=z_f$

Moment-PIC Comparison for FTR



Symbols –PIC (WARP)
o -KV
+ Semi-Gaussian
Lines – Moment Eqs.

A. V. Burov and S. Nagaitsev, Technical Report No. FERMILAB-TM- 2114, 2000



Linearization to Calculate Gradient in Parameter Space - *a*

Base case: $$\frac{d}{dz}\mathbf{X} = \mathbf{O}_{magnets}(z|\mathbf{a})\cdot\mathbf{X}(z)+\mathbf{O}_{SpCh}(\mathbf{X})\cdot\mathbf{X}(z)$$

$$\mathbf{a}\rightarrow\mathbf{a}+\delta\mathbf{a}$$

Direct variation:

$$\frac{d}{dz}\delta\mathbf{X}^{(X)} = \left[\mathbf{O}_{magnets}+\mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(X)}+\left(\delta\mathbf{a}\cdot\frac{\partial}{\partial\mathbf{a}}\mathbf{O}_{magnets}+\delta\mathbf{X}^{(X)}\cdot\frac{\partial}{\partial\mathbf{X}}\mathbf{O}_{SpCh}(\mathbf{X})\right)\cdot\mathbf{X}(z)$$

$$0=\left.\delta\mathbf{X}^{(X)}\right|_{z_i}$$

Change in FoM: $$\delta F=\left.\delta\mathbf{X}^{(X)}\right|_{z_f}\cdot\frac{\partial F}{\partial\mathbf{X}}$$

Adjoint equation: $$\frac{d}{dz}\delta\mathbf{X}^{(Y)} = \left[\mathbf{O}_{magnets}+\mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(Y)}+\delta\dot{\mathbf{X}}^{(Y)}$$

TBD

Final condition: $$\left.\delta\mathbf{X}^{(Y)}\right|_{z_f}$$ ← TBD

Adjoint Magic – Can Show…

Adjoint equation: $$\frac{d}{dz}\delta\mathbf{X}^{(Y)} = \left[\mathbf{O}_{magnets} + \mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(Y)} + \delta\dot{\mathbf{X}}^{(Y)}$$

Integrated backward in z

F – Figure of Merit

Final condition: $$\left.\delta\mathbf{X}^{(Y)}\right|_{z_f}\cdot\hat{\mathbf{J}} = \frac{\partial F}{\partial\mathbf{X}}$$

$\hat{\mathbf{J}} =$ Matrix of zeros and ones

$$\delta F = \int_{z_i}^{z_f} dz\,\delta\mathbf{X}^{(Y)}\cdot\delta\mathbf{a}\cdot\frac{\partial}{\partial\mathbf{a}}\mathbf{O}_{magnets}\cdot\mathbf{X}(z)$$

Base case

Change in FOM

Change in Magnet Parameters

Sensitivity function

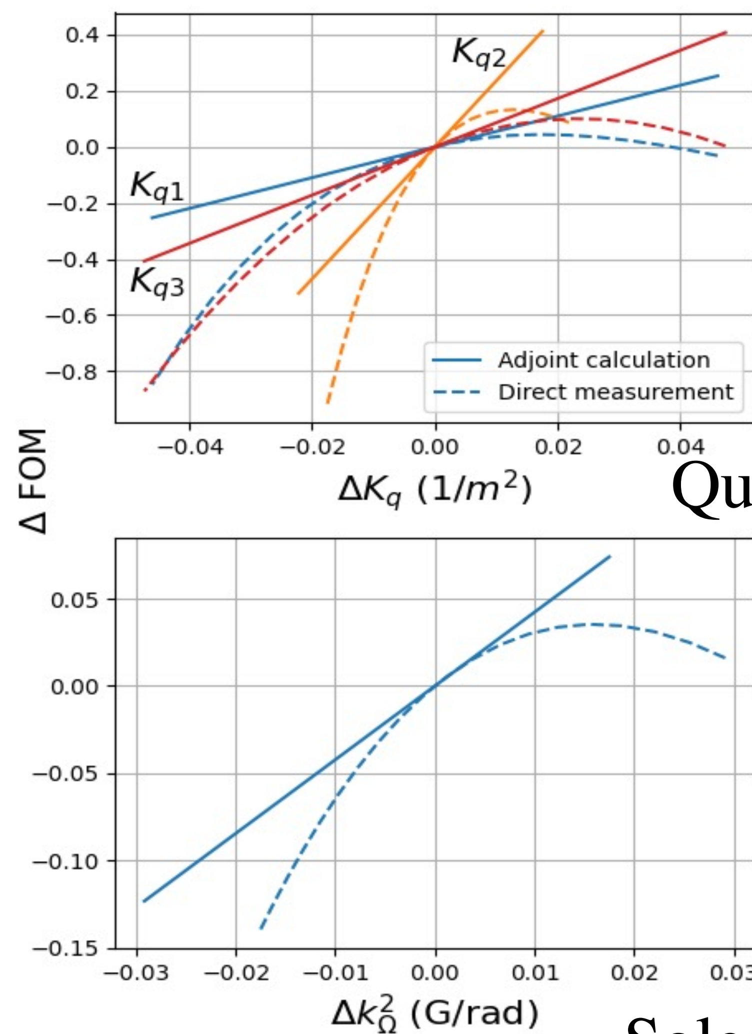
Figure of Merit and Gradient

$$FoM = \frac{1}{2} \sum_{Terms-i} T_i$$

T_1 - beam is round
T_2 – radius is locally constant
T_3 – velocity space is isotropic
T_4 – radial force balance
T_5 – rigid rotation

<u>11 Parameters</u>

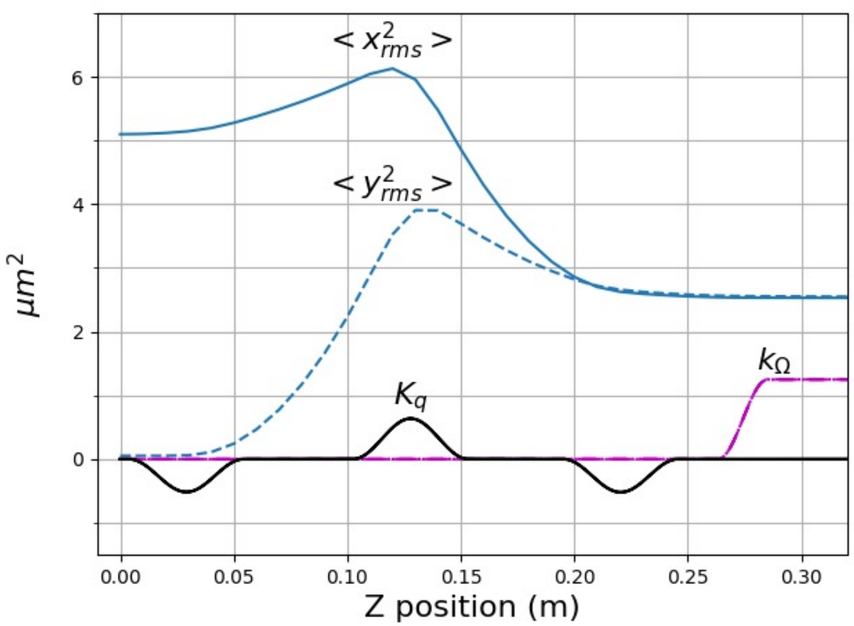
Location of 4 magnets
Strength of 4 magnets
Orientation of 3 Quads

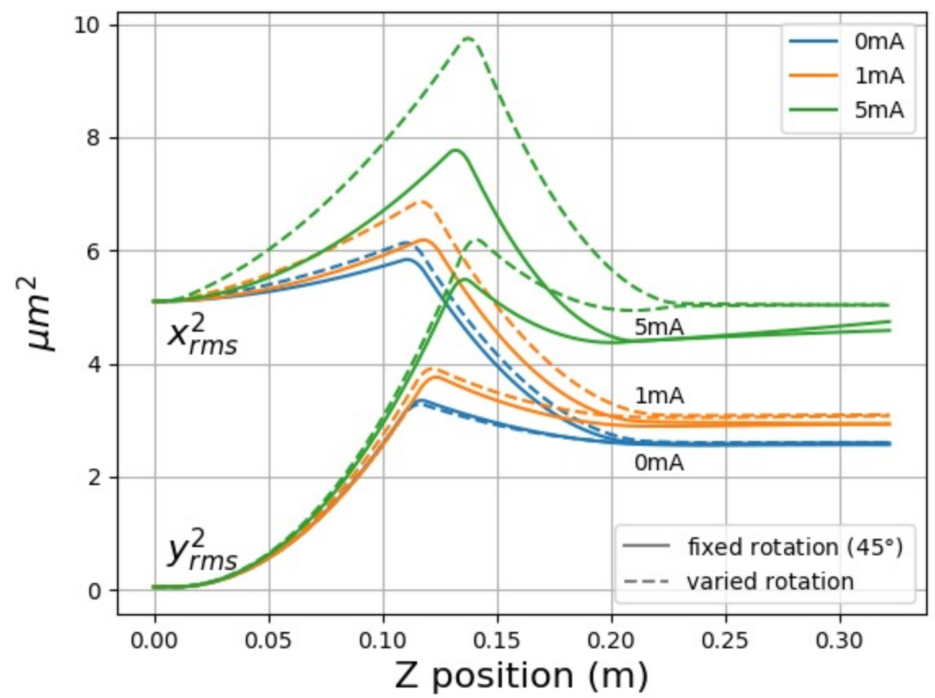


Quad strength

Solenoid strength

Optimization Results



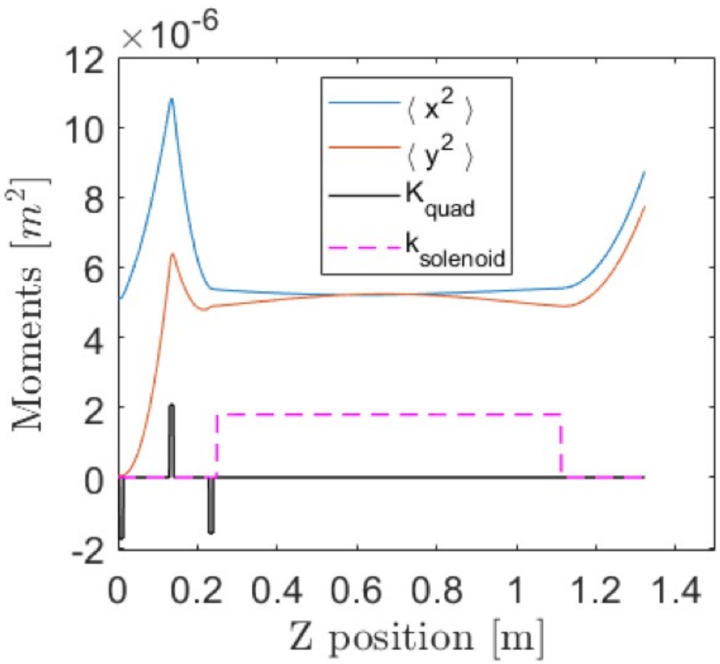


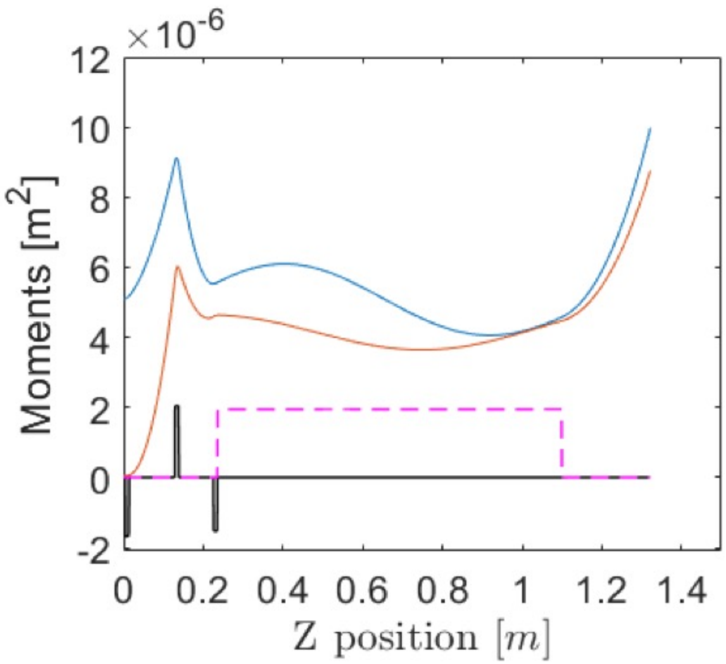
Continuous magnetic field profiles

Variable magnet orientations Required for “strong” space charge

Also S. B. Moroch, et al., arXiv: 2102.13567.

Alternate FoMs





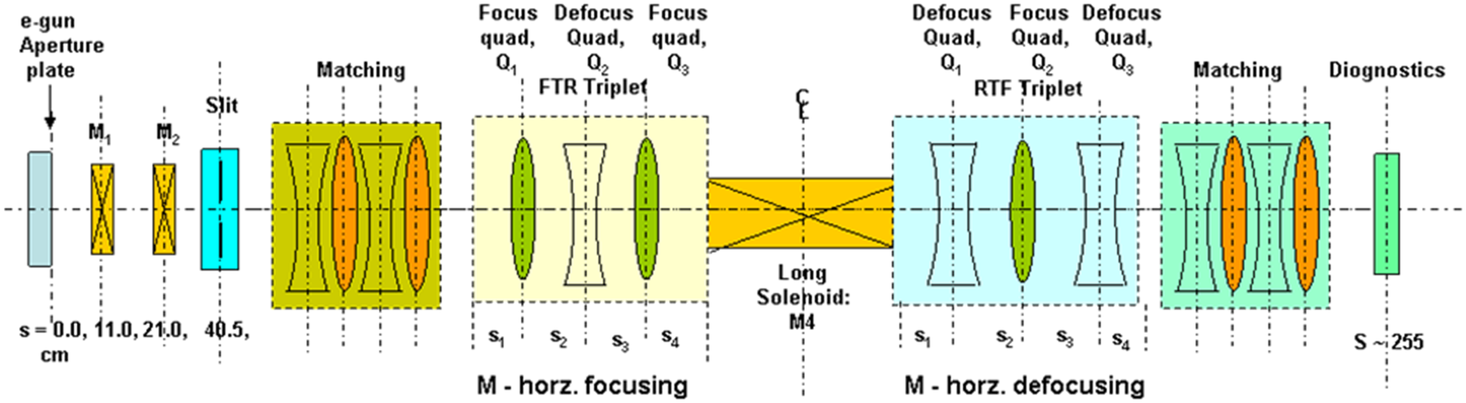
FoM includes radial force balance

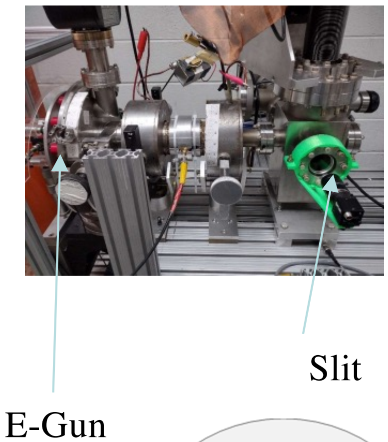
FoM includes transverse energy density in lab frame

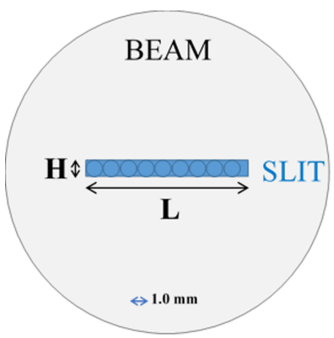
Integrated FoMs also possible

$$F = \int dz\, f(\mathbf{X}, z)$$

Demonstration of Flat/Round Transformations of Angular Momentum and Space Charge Dominated Electron Beams







Generation of highly asymmetric beams

SLIT: L × H	BEAM CURRENT	$\tilde{\varepsilon}_x, \tilde{\varepsilon}_y$	$\tilde{\varepsilon}_x / \tilde{\varepsilon}_y$
10 × 0.2 mm	0.60 mA	53 µm, 1.1 µm	50
10 × 0.5 mm	1.4 mA	53 µm, 2.7 µm	20
10 × 1.0 mm	2.9 mA	53 µm, 5.3 µm	10

Adjoint Relations for Particle Description

TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019)

$$\frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}}$$

Base Solution + Perturbation

$$\left(\mathbf{x}_j, \mathbf{p}_j\right) \rightarrow \left(\mathbf{x}_j, \mathbf{p}_j\right) + \left(\delta \mathbf{x}_j, \delta \mathbf{p}_j\right)$$

$$\rho(\mathbf{x}) \rightarrow \rho(\mathbf{x}) + \delta\rho(\mathbf{x})$$

$$\Phi_T(\mathbf{x}) \rightarrow \Phi_T(\mathbf{x}) + \delta\Phi_T(\mathbf{x})$$

<u>Two Linearized Solutions</u>

$[\delta x_j(t), \delta p_j(t)]$ true

$[\delta \hat{x}_j(t), \delta \hat{p}_j(t)]$ adjoint

$$\delta\Phi_T = \delta\Phi - \frac{v_z}{c}\delta A_z$$ ↔ Includes focusing magnets

Change in symplectic area

Perturbed fields on boundary

$$\sum_j I_j \left(\delta\hat{\mathbf{p}}_j \cdot \delta\mathbf{x}_j - \delta\mathbf{p}_j \cdot \delta\hat{\mathbf{x}}_j \right)\Big|_0^L = -q\varepsilon_0 \int_S da\mathbf{n} \cdot \left[\delta\Phi_T \nabla \delta\hat{\Phi}_T - \delta\hat{\Phi}_T \nabla \delta\Phi_T \right]$$

Adjoint Treatment of Particle Equations

Change in symplectic area

Perturbed fields on boundary

$$\sum_j I_j \left(\delta\hat{\mathbf{p}}_j \cdot \delta\mathbf{x}_j - \delta\mathbf{p}_j \cdot \delta\hat{\mathbf{x}}_j \right)\Big|_0^L = -q\varepsilon_0 \int_S da\,\mathbf{n} \cdot \left[\delta\Phi_T \nabla \delta\hat{\Phi}_T - \delta\hat{\Phi}_T \nabla \delta\Phi_T \right]$$

Pick: $\left(\delta\hat{\mathbf{p}}_j, \delta\hat{\mathbf{x}}_j \right)_{z=L}$

Integrate backward in z

$$\delta F = \sum_j \left(\frac{\partial F}{\partial \mathbf{x}_j} \cdot \delta\mathbf{x}_j - \delta\mathbf{p}_j \cdot \left(-\frac{\partial F}{\partial \mathbf{p}_j} \right) \right)\Bigg|_L = -q\varepsilon_0 \int_S da\,\mathbf{n} \cdot \left[\delta\Phi_T \nabla \delta\hat{\Phi}_T \right]$$

Change in FoM

Arb. $F(\boldsymbol{x}, \mathbf{p}, \mathbf{z}_f)$

Change in focusing magnets.

Includes multipole variation

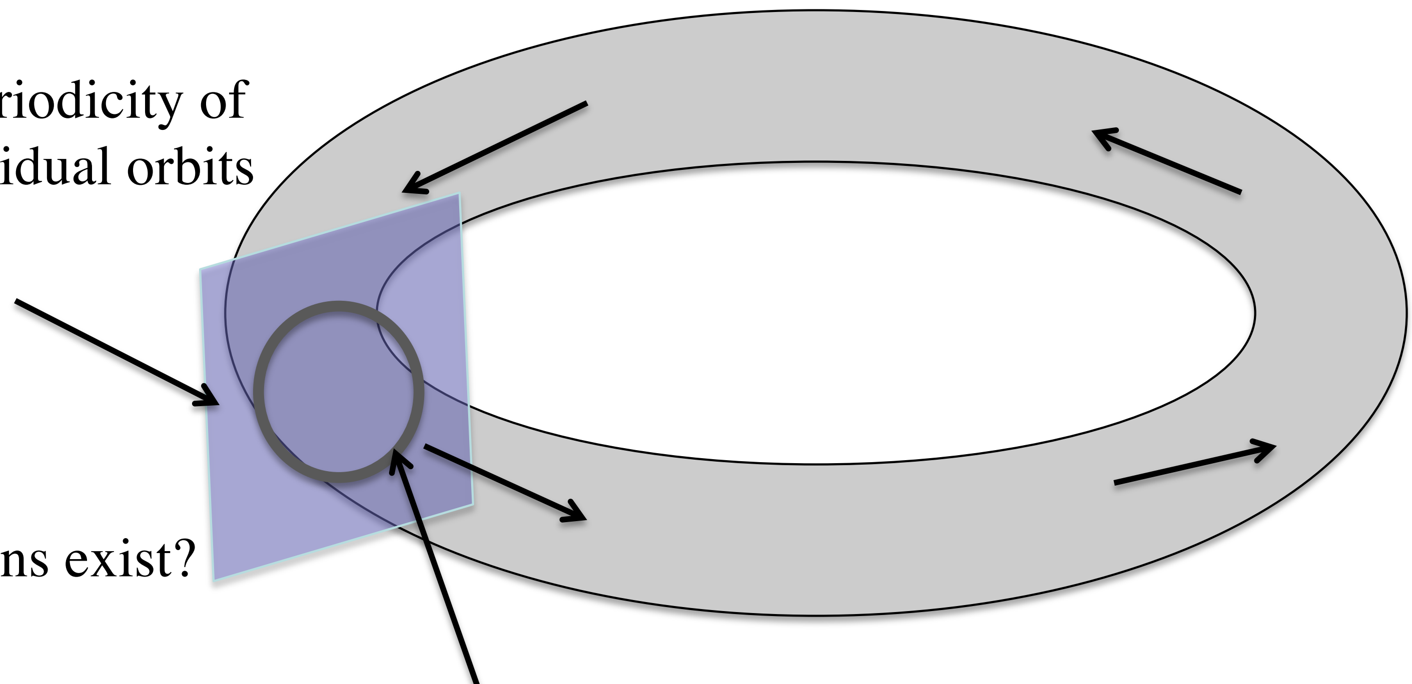
Sensitivity

Circular Accelerators-Periodicity?

Particles return to initial plane.

Solve Eqs. of motion and self fields

Desire to maintain periodicity of
distribution, not individual orbits



Start here with 4D particle coordinates

Problems:
Do periodic distributions exist?
Most likely no.

Represent DF in reduced basis.
Make basis coefficients periodic.

Optimize FoM subject to periodicity

Constrained Optimization
“Adjoint with a Chaser”

Summary slide, 5th ICFA mini-workshop on Space Charge
Theme: Bridging the gap in space charge dynamics

Adjoint methods are a powerful way to evaluate parameter dependences in many systems involving charged particle dynamics.

Application to 2nd-moments has been implemented.
Need to develop applications to particle distributions.
Optimize phase space distributions, circular Accelerators.

Adjoint relation is a consequence of retaining self-fields.

Theory Perspective: Space Charge – Good.

Moment Equations

Transverse phase space: $x, x' = \frac{dx}{dz}, y, y' = \frac{dy}{dz}$

The second order moments are averages of the 4 by 4 elements of the Sigma matrix.

$$\underline{\underline{\Sigma}} = \begin{bmatrix} xx & xx' & xy & xy' \\ x'x & x'x' & x'y & x'y' \\ yx & yx' & yy & yy' \\ y'x & y'x' & y'y & y'y' \end{bmatrix}$$

Moments: $\mathbf{Q}, \mathbf{P}, \mathbf{E}, L$

$$\mathbf{Q} = \begin{pmatrix} Q_+ \\ Q_- \\ Q_x \end{pmatrix} = \begin{pmatrix} \langle x^2 + y^2 \rangle / 2 \\ \langle x^2 - y^2 \rangle / 2 \\ \langle xy \rangle \end{pmatrix} \quad \mathbf{P} = \frac{d}{dz}\mathbf{Q} = \begin{pmatrix} P_+ \\ P_- \\ P_x \end{pmatrix} = \begin{pmatrix} \langle xx' + yy' \rangle \\ \langle xx' - yy' \rangle \\ \langle yx' + xy' \rangle \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} E_+ \\ E_- \\ E_x \end{pmatrix} = \begin{pmatrix} \langle x'^2 + y'^2 \rangle \\ \langle x'^2 - y'^2 \rangle \\ 2\langle y'x' \rangle \end{pmatrix}$$

Angular momentum $L = \langle xy' - yx' \rangle$

Linearized System

Base case

$$\frac{d}{dz}\mathbf{Q} = \mathbf{P}$$

$$\frac{d}{dz}\mathbf{P} = \mathbf{E} + \mathbf{O}\cdot\mathbf{Q}$$

$$\frac{d}{dz}\mathbf{E} = \mathbf{O}\cdot\mathbf{P} + \mathbf{N}L$$

$$\frac{d}{dz}L = -\mathbf{N}^{\dagger}\cdot\mathbf{Q}$$

Linear perturbation due to true change in parameters

$$\frac{d}{dz}\delta\mathbf{Q}^{(X)} = \delta\mathbf{P}^{(X)}$$

$$\frac{d}{dz}\delta\mathbf{P}^{(X)} = \delta\mathbf{E}^{(X)} + \mathbf{O}\cdot\delta\mathbf{Q}^{(X)} + \delta\mathbf{O}^{(X)}\cdot\mathbf{Q}$$

$$\frac{d}{dz}\delta\mathbf{E}^{(X)} = \mathbf{O}\cdot\delta\mathbf{P}^{(X)} + \mathbf{N}\delta L^{(X)} + \delta\mathbf{O}^{(X)}\cdot\mathbf{P} + \delta\mathbf{N}^{(X)}L$$

$$\frac{d}{dz}\delta L^{(X)} = -\mathbf{N}^{\dagger}\cdot\delta\mathbf{Q}^{(X)} - \delta\mathbf{N}^{\dagger(X)}\cdot\mathbf{Q}$$

Adjoint system

$$\frac{d}{dz}\delta\mathbf{Q}^{(Y)} = \delta\mathbf{P}^{(Y)}$$

$$\frac{d}{dz}\delta\mathbf{P}^{(Y)} = \delta\mathbf{E}^{(Y)} + \mathbf{O}\cdot\delta\mathbf{Q}^{(Y)}$$

$$\frac{d}{dz}\delta\mathbf{E}^{(Y)} = \mathbf{O}\cdot\delta\mathbf{P}^{(Y)} + \mathbf{N}\delta L^{(Y)} + \delta\dot{\mathbf{E}}^{(Y)}$$

$$\frac{d}{dz}\delta L^{(Y)} = -\mathbf{N}^{\dagger}\cdot\delta\mathbf{Q}^{(Y)}$$

Sensitivity functions

$$\delta FoM = \int_{z_i}^{z_f} dz\left\{\delta\mathbf{P}^{(Y)}\cdot\delta\mathbf{O}_{Q,B}{}^{(X)}\cdot\mathbf{Q} + \delta L^{(Y)}\mathbf{Q}\cdot\delta\mathbf{N}_{Q,B}{}^{(X)} - \delta\mathbf{Q}^{(Y)}\cdot\delta\mathbf{O}_{Q,B}{}^{(X)}\cdot\mathbf{P} - \delta\mathbf{Q}^{(Y)}\cdot\delta\mathbf{N}_{Q,B}{}^{(X)}L\right\}$$

Change in magnet parameters

Figure of Merit and Gradient

Constant radius, Round

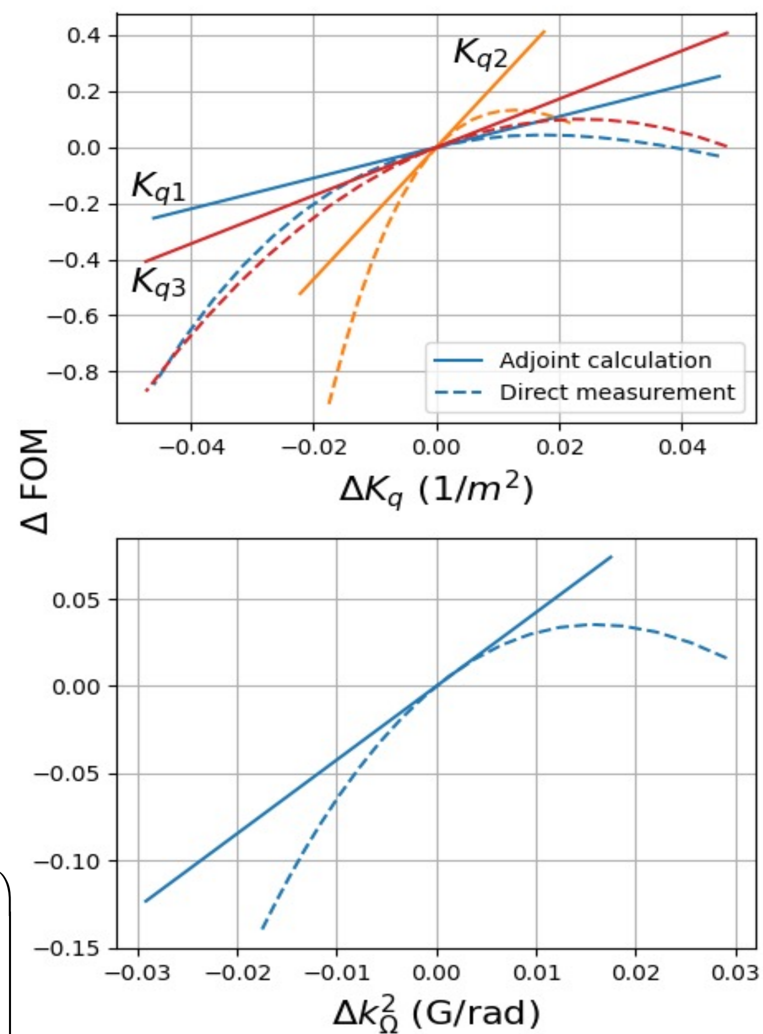
$$F = \frac{1}{2}\left[\left|\mathbf{P}\right|^2 + k_0^2\left(Q_-^2 + Q_x^2\right) + k_0^{-2}\left(E_-^2 + E_x^2\right)\right]$$

$$+\frac{1}{2}\left[k_0^{-2}\left(E_+ - \frac{1}{2}k_\Omega^2 Q_+ + \Lambda\right)^2 + \left(2E_+Q_+ - L^2\right)^2\right]$$

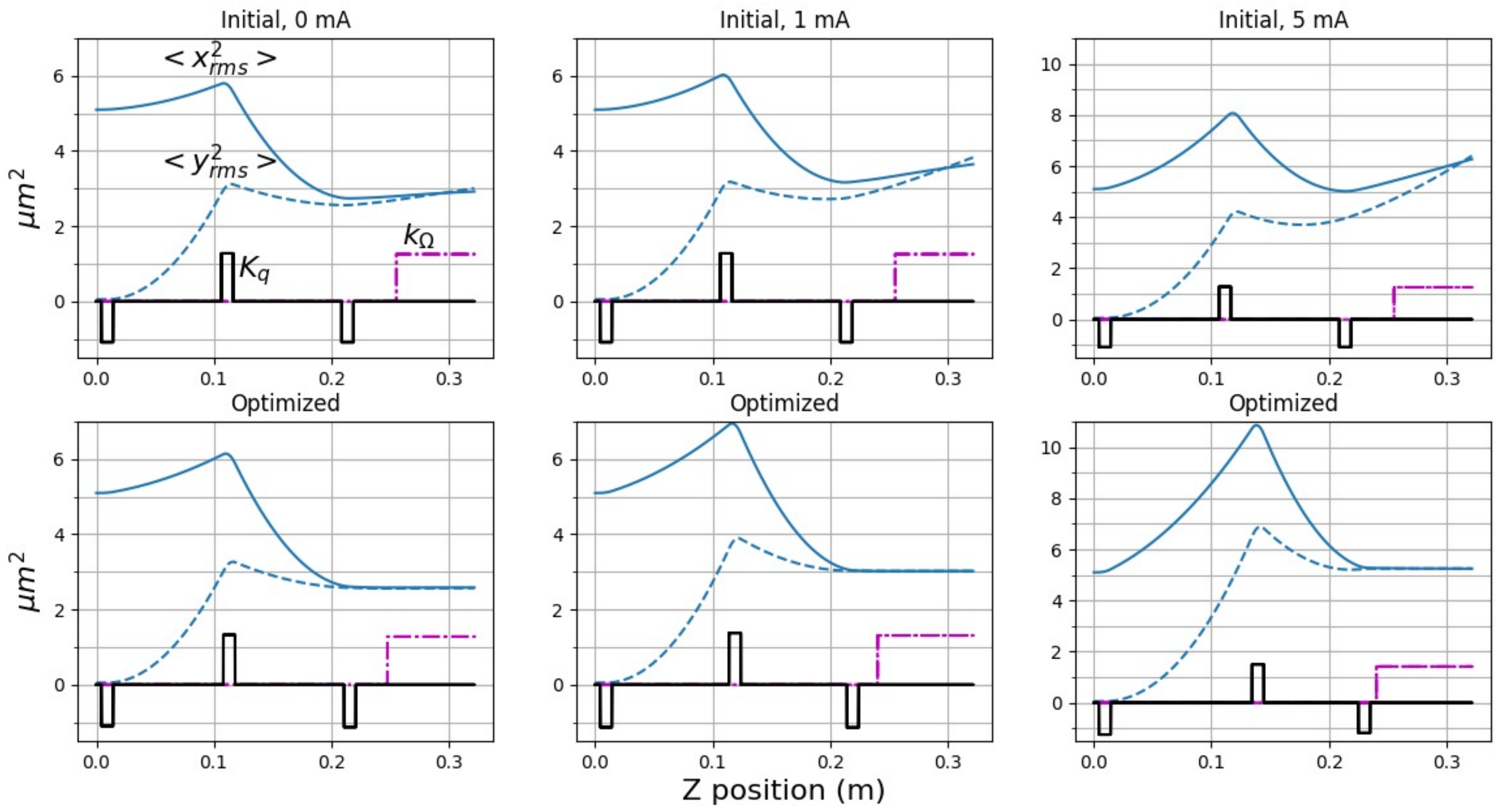
Radial force balance, Rigid rotation

$$L = \left\langle xy' - yx' \right\rangle$$

$$\mathbf{Q} = \begin{pmatrix} \frac{1}{2}\left\langle x^2 + y^2 \right\rangle \\ \frac{1}{2}\left\langle x^2 - y^2 \right\rangle \\ \left\langle xy \right\rangle \end{pmatrix} \quad \mathbf{P} = \begin{pmatrix} \left\langle xx' + yy' \right\rangle \\ \left\langle xx' - yy' \right\rangle \\ \left\langle yx' + xy' \right\rangle \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} \left\langle x'^2 + y'^2 \right\rangle \\ \left\langle x'^2 - y'^2 \right\rangle \\ 2\left\langle y'x' \right\rangle \end{pmatrix}$$



Optimization – Space Charge Compensation



Global Beam Sensitivity Function for Electron Guns

Goal

Derive and Calculate a function that gives the variation of specific beam parameters to

- variations in electrode potential/position
- variations in magnet current/position

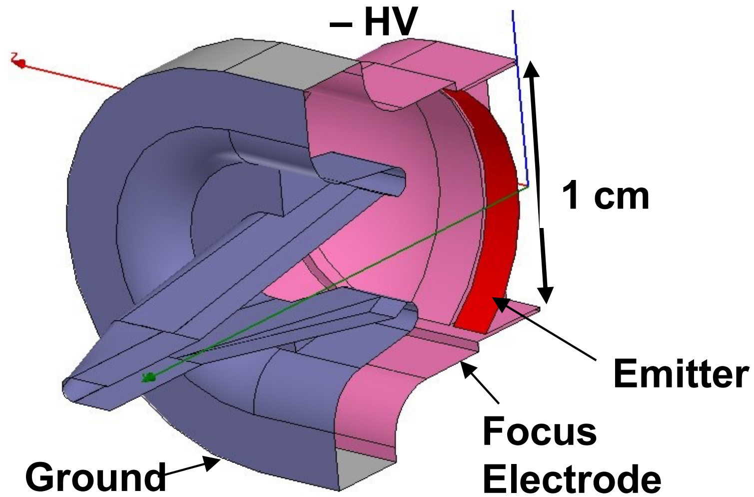
Can be used to

- establish manufacturing tolerances
- optimize gun designs

Should be embedded in gun code (e.g. Michelle)

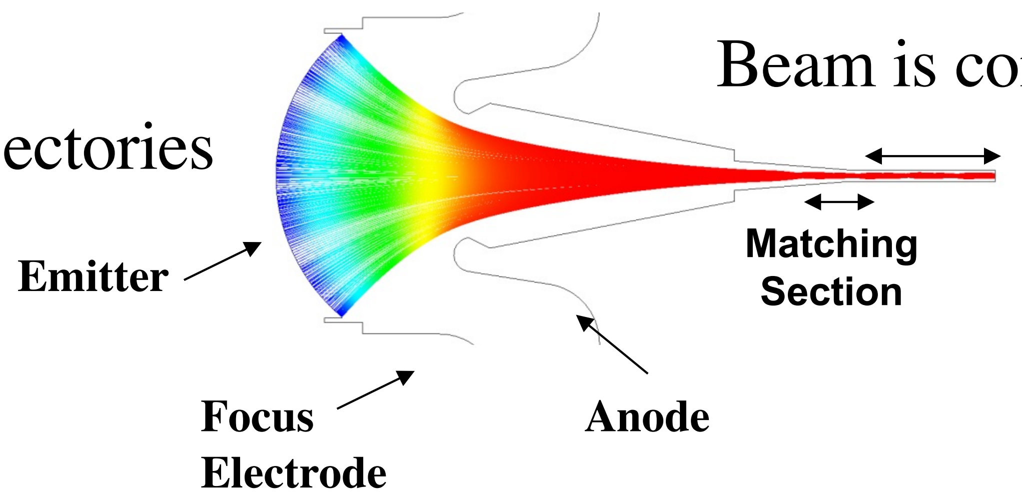
Thermionic Cathode Electron Gun

Solid Model of Electrodes

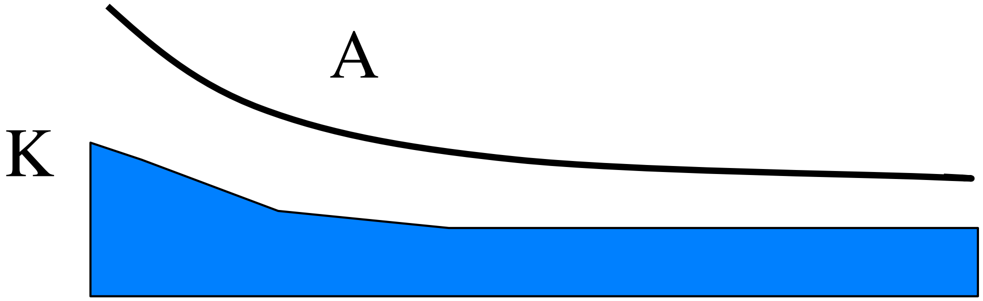


Cut away view of trajectories

Beam is compressed



What shape to make electrodes?



Michelle: Petillo, J; Eppley, K; Panagos, D; et al., IEEE TPS 30, 1238-1264 (2002).

Code (Michelle) solves the following equations:

Equations of motion for N particles j=1,N

$$\frac{d\boldsymbol{x}_j}{dt} = \frac{\partial H}{\partial \boldsymbol{p}} \qquad \frac{d\boldsymbol{p}_j}{dt} = -\frac{\partial H}{\partial \boldsymbol{x}}$$

Start with vacuum fields

Accumulates a charge density

$$\rho(\boldsymbol{x}) = \sum_j I_j \int_0^{T_j} dt\ \delta(\boldsymbol{x} - \boldsymbol{x}_j(t))$$

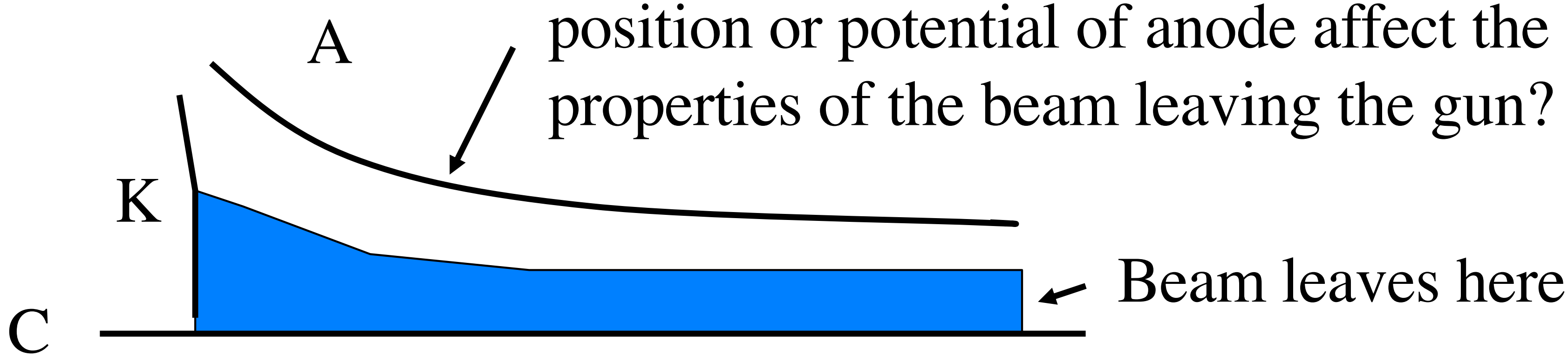
Solves Poisson Equation

$$-\nabla^2 \Phi = \rho / \varepsilon_0$$

Iterates until converged

Sensitivity Function

<u>Basic question:</u> How do small changes in position or potential of anode affect the properties of the beam leaving the gun?



<u>Direct Approach</u>:

Do many simulations with different anode potentials, positions

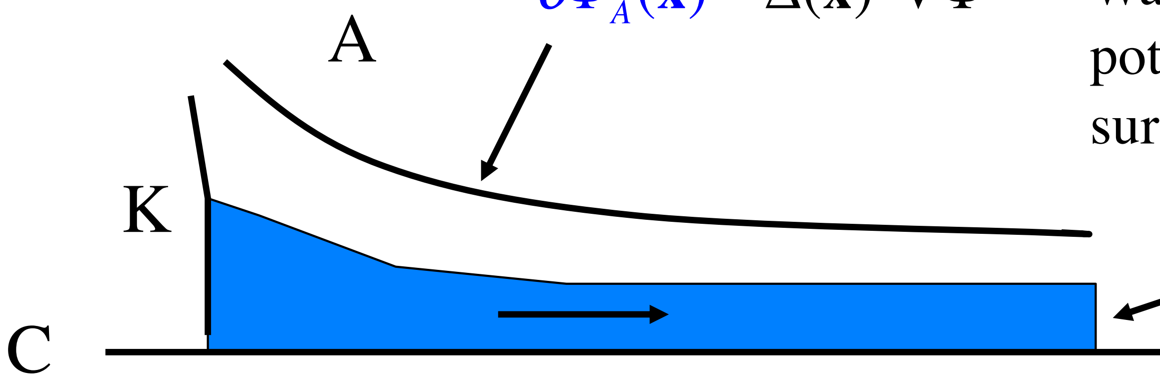
Select the best based on some metric measured at the exit.

Adjoint Problem

Problem #1

$\delta\Phi_A(\mathbf{x}) = \Delta(\mathbf{x})\cdot\nabla\Phi$

Wall displacement changes potential on unperturbed surface.



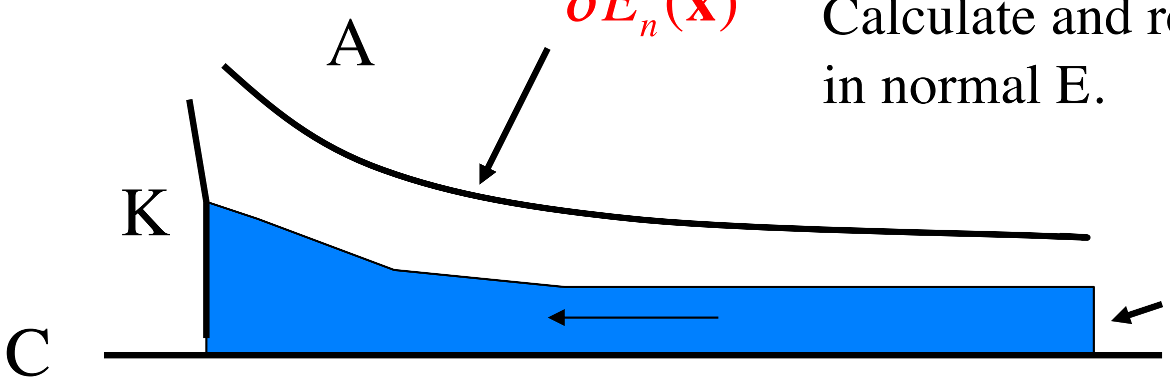
Leads to change in RMS beam radius ΔR_{RMS}

Problem #2

x

$\delta E_n(\mathbf{x})$

Calculate and record change in normal E.



Reverse and perturb electron coordinates

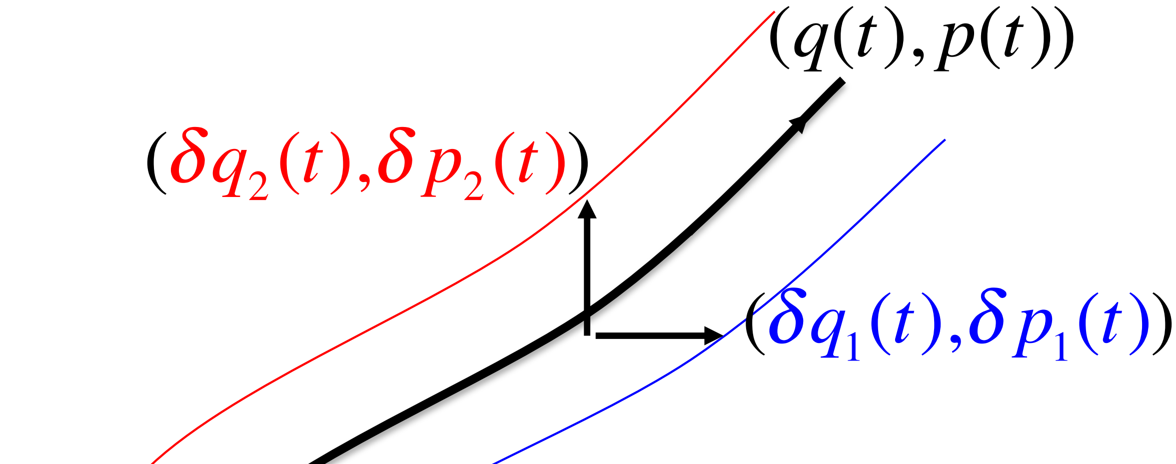
Electrons run backwards

Sensitivity function

δE_n Is the sensitivity function

$$\Delta R_{RMS} \propto \int_S da\, \delta\Phi_A(\mathbf{x})\delta E_n(\mathbf{x})$$

Hamilton's Equations $H(\boldsymbol{p},\boldsymbol{q},t)$ Conserve Symplectic Area



$$\frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}}$$

perturbed orbit #1

$$\frac{d\delta\mathbf{q}_1}{dt} = \frac{\partial^2 H}{\partial\mathbf{p}\,\partial\mathbf{q}}\cdot\delta\mathbf{q}_1 + \frac{\partial^2 H}{\partial\mathbf{p}\,\partial\mathbf{p}}\cdot\delta\mathbf{p}_1$$

$$\frac{d\delta\mathbf{p}_1}{dt} = -\frac{\partial^2 H}{\partial\mathbf{q}\,\partial\mathbf{q}}\cdot\delta\mathbf{q}_1 - \frac{\partial^2 H}{\partial\mathbf{q}\,\partial\mathbf{p}}\cdot\delta\mathbf{p}_1$$

perturbed orbit #2

$$\frac{d\delta\mathbf{q}_2}{dt} = \ldots$$

$$\frac{d\delta\mathbf{p}_2}{dt} = -\ldots$$

$$\frac{d}{dt}\left(\delta\mathbf{p}_1\cdot\delta\mathbf{q}_2 - \delta\mathbf{p}_2\cdot\delta\mathbf{q}_1\right) = 0$$

Area conserved for any choice of 1 and 2

Reference Solution + Two Linearized Solutions

$$\left(\mathbf{x}_j,\mathbf{p}_j\right)\rightarrow\left(\mathbf{x}_j,\mathbf{p}_j\right)+\left(\delta\mathbf{x}_j,\delta\mathbf{p}_j\right)$$

$$\rho(\mathbf{x})\rightarrow\rho(\mathbf{x})+\delta\rho(\mathbf{x})$$

$$\Phi(\mathbf{x})\rightarrow\Phi(\mathbf{x})+\delta\Phi(\mathbf{x})$$

Reference Solution   Perturbation

<u>Two Linearized Solutions</u>

$[\delta x_j(t),\delta p_j(t)]$ true

$[\delta\hat{x}_j(t),\delta\hat{p}_j(t)]$ adjoint

subject to different BC's

Can show

$$\sum_j I_j\left(\delta\hat{\mathbf{p}}_j\cdot\delta\mathbf{x}_j-\delta\mathbf{p}_j\cdot\delta\hat{\mathbf{x}}_j\right)\Big|_0^{T_j}=-q\varepsilon_0\int_S da\,\mathbf{n}\cdot\left[\delta\Phi\nabla\delta\hat{\Phi}-\delta\hat{\Phi}\nabla\delta\Phi\right]$$

Generalized Green Theorem

Generalized Green's Theorem

$$\sum_j I_j \left(\delta\hat{\mathbf{p}}_j \cdot \delta\mathbf{x}_j - \delta\mathbf{p}_j \cdot \delta\hat{\mathbf{x}}_j \right)\Big|_0^{T_j} = -q\varepsilon_0 \int_S da\,\mathbf{n} \cdot \left[\delta\Phi \nabla \delta\hat{\Phi} - \delta\hat{\Phi} \nabla \delta\Phi \right]$$

Problem #1 (true problem) Unperturbed trajectories at cathode, Perturbed potential on boundary.

$$\delta p_j\big|_0 = 0, \quad \delta x_j\big|_0 = 0, \quad \delta\Phi(\mathbf{x}) \neq 0$$

Problem #2 (adjoint problem) Perturbed trajectories at exit, Unperturbed potential on boundary.

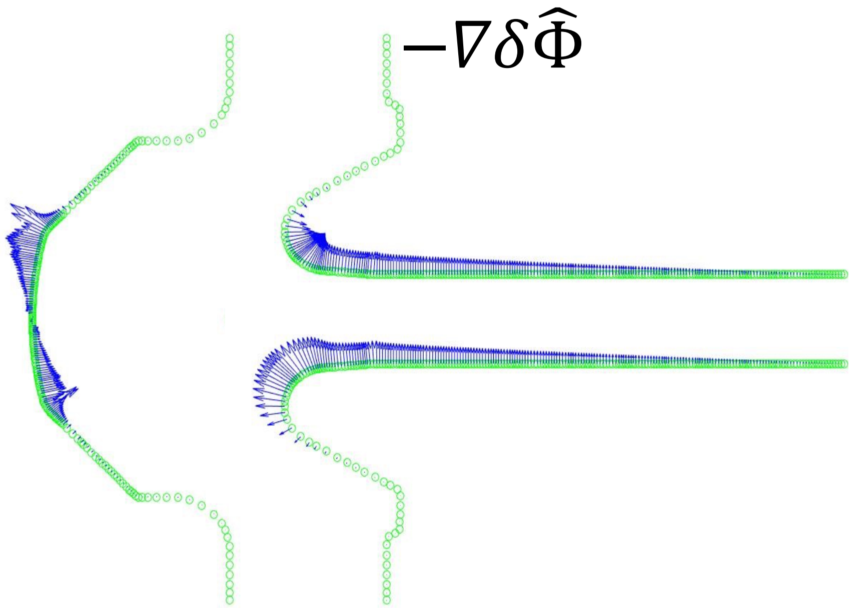
$$\boxed{\delta \hat{p}_j\big|_T = \lambda \mathbf{x}_{\perp j},} \quad \delta x_j\big|_T = 0, \quad \delta\hat{\Phi}(\mathbf{x}) = 0$$

$$\lambda I R_{RMS} \delta R_{RMS} = \lambda \sum_j I_j \left(\mathbf{x}_j \cdot \delta\mathbf{x}_j \right)\Big|_{T_j} = -q\varepsilon_0 \int_S da\,\delta\Phi \left(\mathbf{n} \cdot \nabla \delta\hat{\Phi} \right)$$

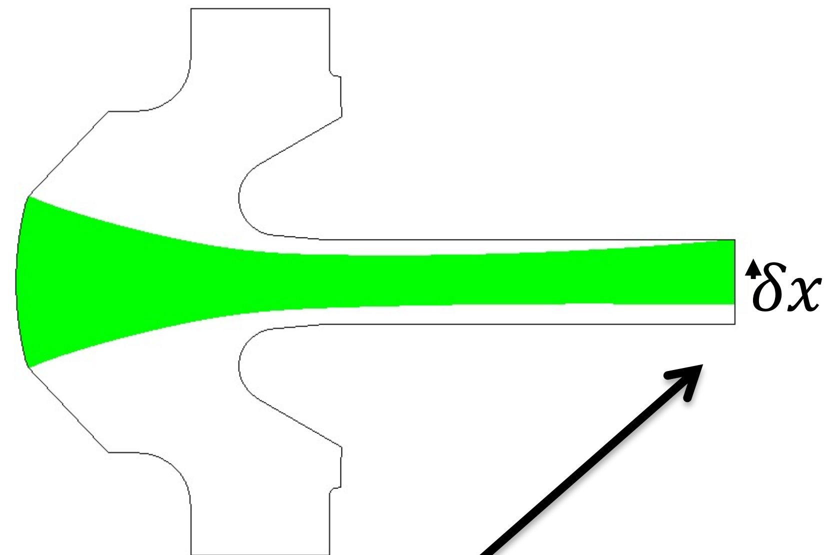
Sensitivity Function

Vertical Displacement of the Beam

Vector plot of the ‘sensitivity’ or Green’s function



‘Direct’ MICHELLE Simulation with Perturbed Anode Voltages



$$\delta x = -\frac{q\varepsilon_0}{\lambda I}\int_S da\,\mathbf{n}\cdot\delta\Phi\nabla\delta\hat{\Phi}$$

Predicted displacement / Calculated displacement = 0.9969

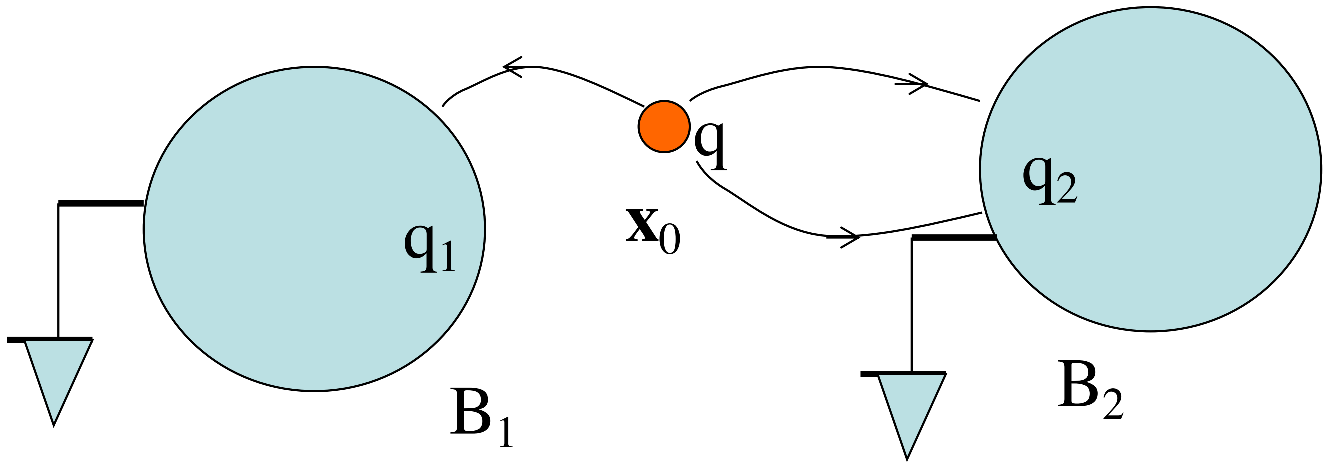
Recent Adjoint Approaches

- Beam optics sensitivity function, TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019); doi: 10.1063/1.5079629
- Stellarator Optimization and Sensitivity, E. Paul, M. Landreman, TMA, *J. Plasma Phys*. (2019), v*ol*. 85, 905850207, *J. Plasma Phys*. (2021), v*ol*. 87, 905870214
- Optimization of Flat to Round Transformers in Particle Accelerators, L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and TMA, Phys Rev Accel and Beams V25, 044002 (2022).
- Adjoint Equations for Beam-Wave Interaction and Optimization of TWT Design, A. Vlasov, TMA, D. Chernin and I. Chernyavskiy, IEEE Trans. Plasma Sci. V. March (2022).

Give a child a hammer and everything becomes a nail.

Basic Adjoint Example
Jackson, Classical Electrodynamics
Problems 1.12 and 1.13

A charge q is placed at an arbitrary point, $\mathbf{x}_o$, relative to two grounded, conducting electrodes.



What is the charge q_1 on the surface of electrode 1?

<u>Repeat for different $\mathbf{x_0}$</u>

Solution – Green's Reciprocation Theorem

Prob #1
Direct evaluation

$$\nabla^2\phi = -q\delta(\mathbf{x}-\mathbf{x}_0) \quad \text{BC:} \quad \phi\big|_{B1} = \phi\big|_{B2} = \phi(x\to\infty) = 0$$

$$q_1 = \int_{B1} d^2x\,\mathbf{n}\cdot\nabla\phi$$

Repeated for each x_0

Prob #2
Adjoint Problem

$$\nabla^2\psi = 0 \quad \text{BC:} \quad \psi\big|_{B1} = 1, \quad \psi\big|_{B2} = \psi(x\to\infty) = 0$$

Done once

Green's Theorem

$$\int_V d^3x\left(\psi\nabla^2\phi - \phi\nabla^2\psi\right) = \int_S d^2x\, n\cdot\left(\psi\nabla\phi - \phi\nabla\psi\right)$$

($\phi\nabla^2\psi \to 0$, $\phi\nabla\psi \to 0$)

When the dust settles:

$$-q\psi(\mathbf{x}_0) = q_1$$