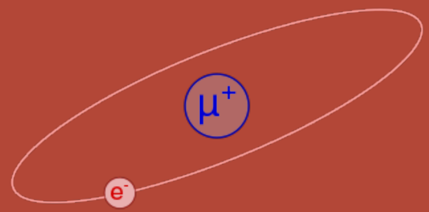
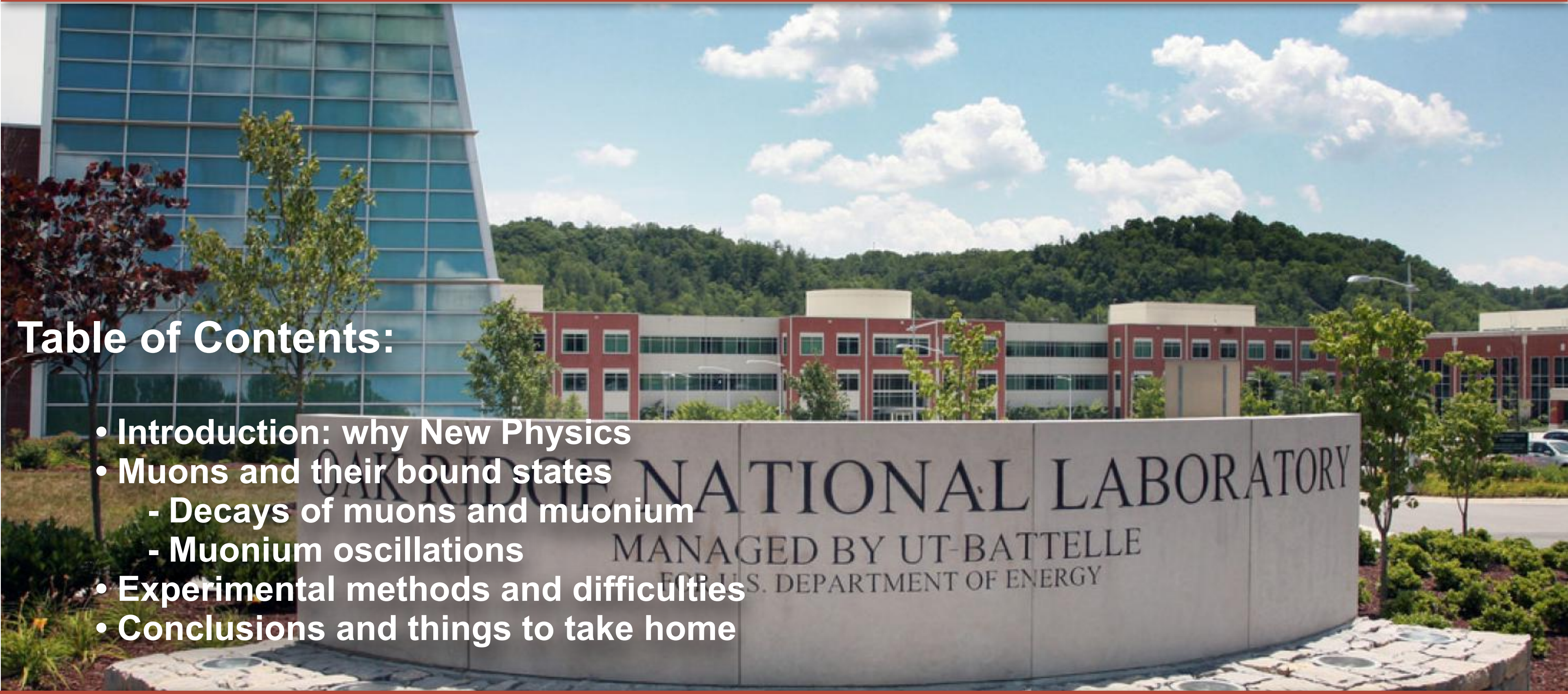
Low-Energy Muon Physics

Theory





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New Science and Applications at the SNS,
Oak Ridge National Lab, 6/10-11/2026

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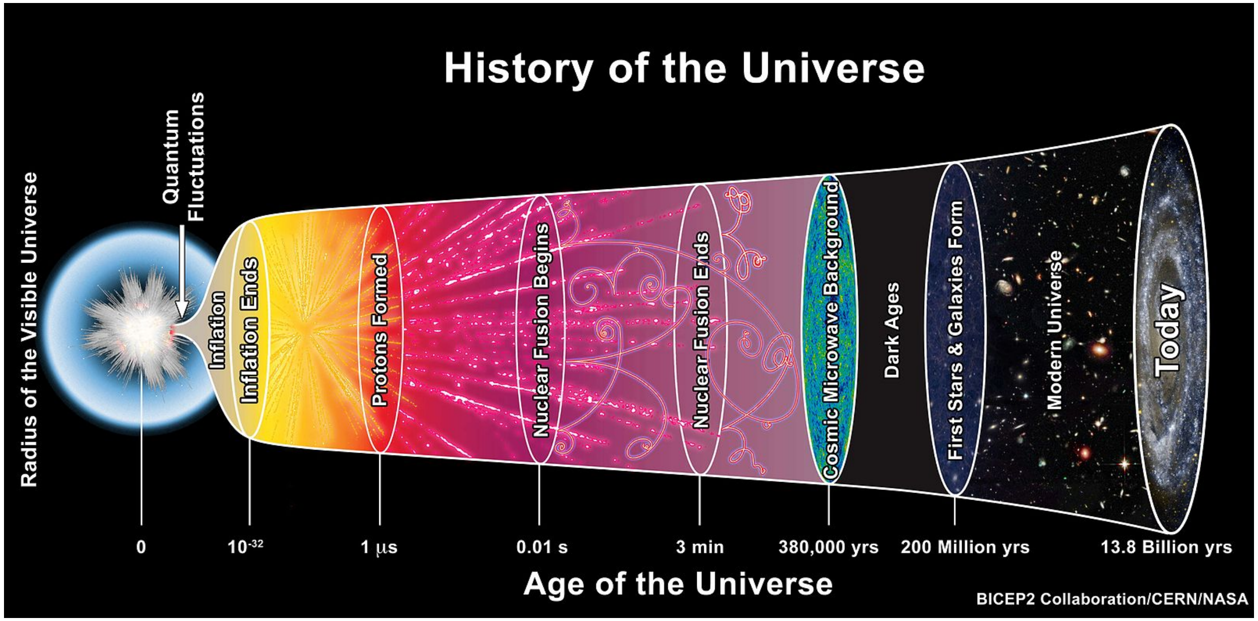
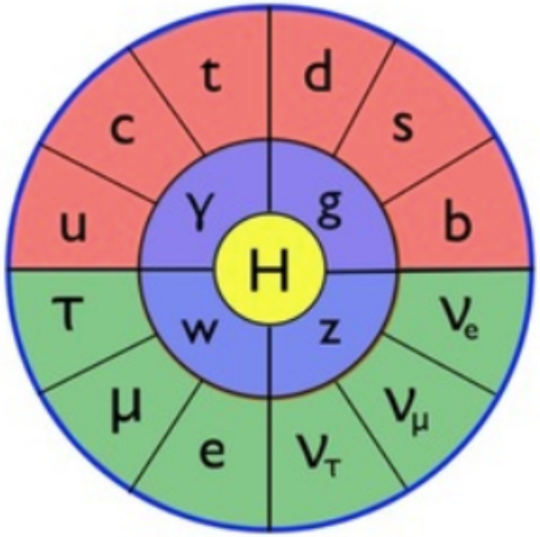
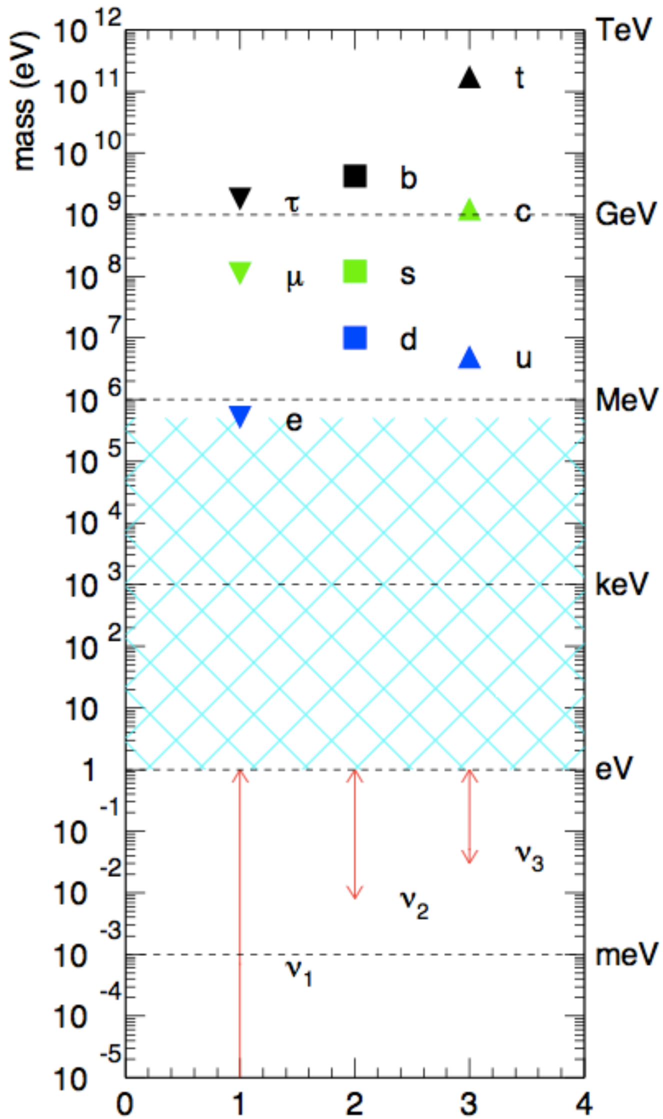
★ Is it possible to build the Universe using the Standard Model as a tool?
- no, but maybe it can tell us where to look for new tools





★ The era of "guaranteed discoveries" is over (top quark, electroweak breaking)
- new experiments designed to study rare decays or perform precision studies of various processes might point us in the right direction

Fundamental physics with muons: flavor problem



★ SM and BSM Flavor problem

★ Flavor problem: patterns of masses of particles

- quarks

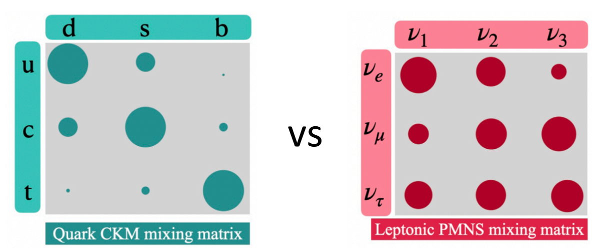
$$\frac{m_d}{m_u} \simeq 2 , \quad \frac{m_s}{m_d} \simeq 21 ,$$
$$\frac{m_t}{m_c} \simeq 267 , \quad \frac{m_c}{m_u} \simeq 431 , \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5 .$$

- leptons

$$\frac{m_\tau}{m_\mu} \simeq 17 , \quad \frac{m_\mu}{m_e} \simeq 207 .$$

★ Flavor problem: pattern of fermion mixing

- why is the quark mixing matrix so different from the neutrino mixing matrix?



S. Cao, et al.

★ Flavor problem: nature of neutrino mass?

Fundamental physics with muons: flavor problem

★ Flavor problem: flavor-changing neutral currents (FCNC)

- there is no term in the SM Lagrangian that leads to FCNC effects: quantum effects (one loop process)
- quarks: massive quarks and non-zero mixing parameters automatically lead to FCNC processes: $b \to s\gamma, c \to u\ell\bar{\ell}, B^0 - \bar{B}^0$-mixing, etc.
- leptons: massive neutrinos and non-zero mixing parameters automatically lead to FCNC processes: $\mu \to e\gamma, \mu \to eee, \mu A \to eA$, etc.

★ Flavor problem: patterns of masses of particles and neutrino mass: new symmetry?

- there could be a mechanism generating mass patterns (Froggatt-Nielsen, etc.)…

A. Blechman, AAP, G.K. Yeghiyan

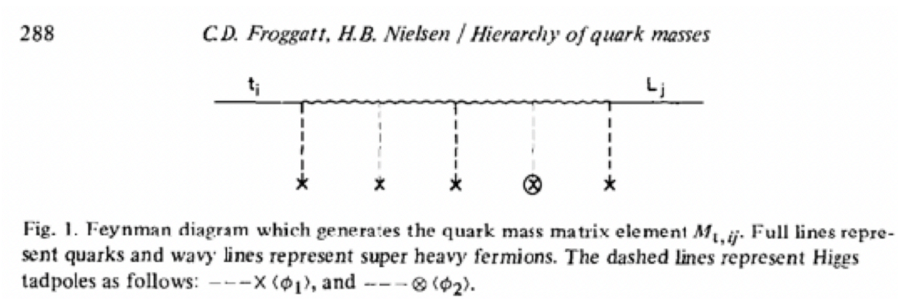
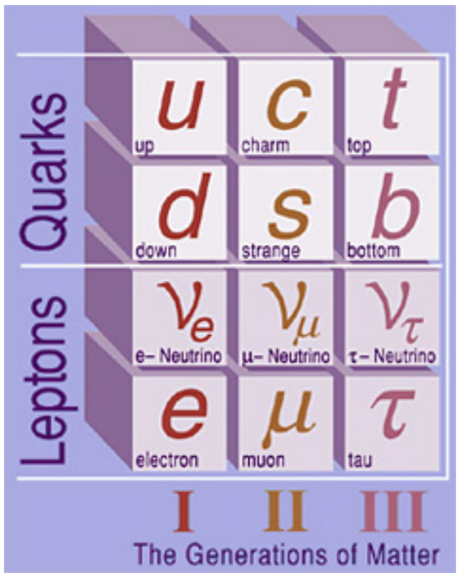

288 *C.D. Froggatt, H.B. Nielsen / Hierarchy of quark masses*

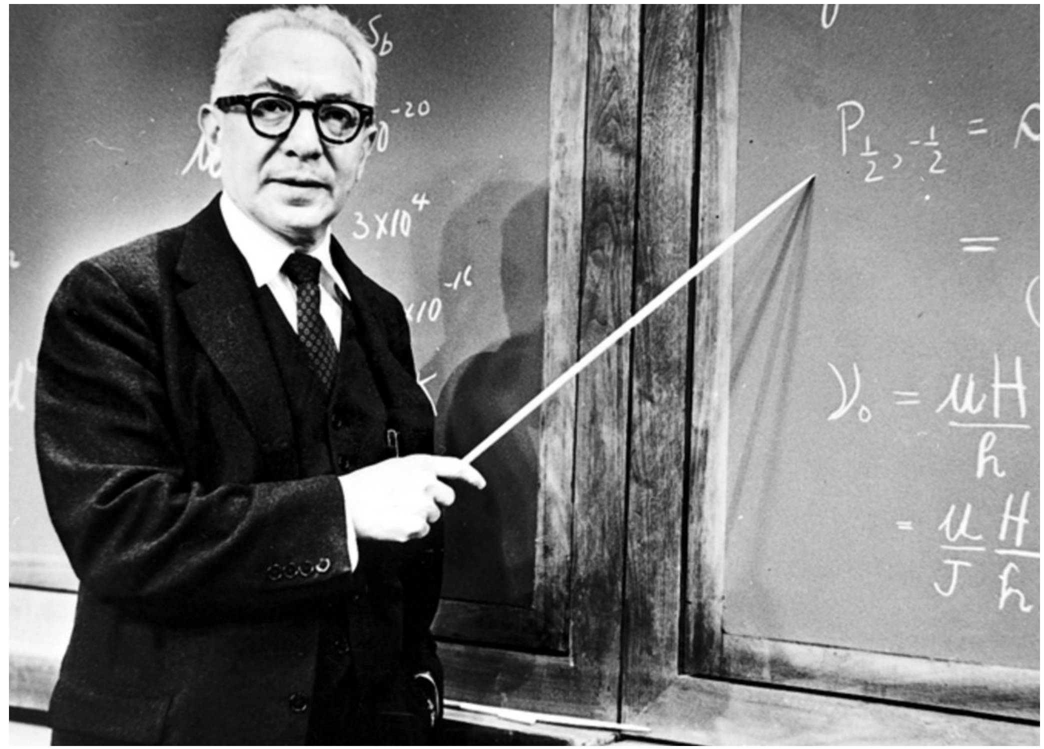
Fig. 1. Feynman diagram which generates the quark mass matrix element $M_{t,ij}$. Full lines represent quarks and wavy lines represent super heavy fermions. The dashed lines represent Higgs tadpoles as follows: $---\times\ \langle\phi_1\rangle$, and $---\otimes\ \langle\phi_2\rangle$.

- … or maybe not (a “just so” solution?)

Muons as tools to discover New Physics?

★ Muon: first particle of the second generation discovered (1936) - 90 years ago!





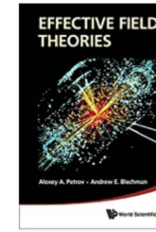
“Who ordered that?” I.I. Rabi

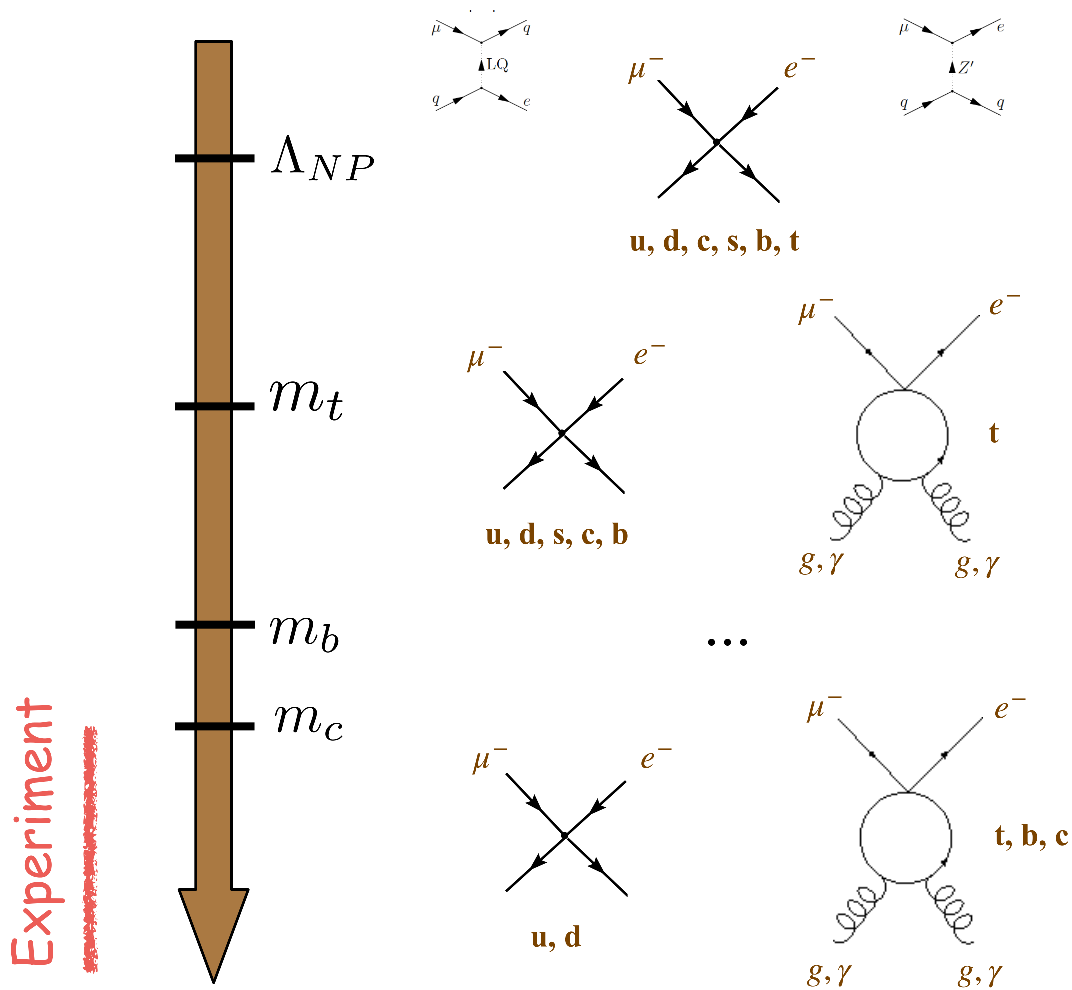
Can we study fundamental physics with muons?

Flavor violation and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand ALL relevant energy scales for the problem at hand





New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

Flavor violation and effective Lagrangians

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59) of $Q_i^{(6)}$ operators

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and ψ^2H^3.

X^3		H^6 and H^4D^2		ψ^2H^3 + h.c.	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, ψ^2XH, and ψ^2H^2D.

X^2H^2		ψ^2XH + h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p\gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p\tau^I\gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{H} G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p\gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\tau^I\widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p\gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p\tau^I\gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A d_r\right)H G_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p\gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p\gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)H B_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p\gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{L}_s\gamma^\mu L_t\right)$	Q_{ee}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$	Q_{le}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{uu}	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{lu}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p\gamma^\mu\tau^I Q_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p\gamma^\mu d_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{eu}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{qe}	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p\gamma^\mu\tau^I L_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{ed}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
		$Q_{ud}^{(1)}$	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p\gamma^\mu T^A q_r\right)\left(\overline{u}_s\gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p\gamma^\mu T^A u_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p\gamma^\mu q_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p\gamma^\mu T^A Q_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left((\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left((\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left((\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left((\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left((\overline{L}_p^j\sigma_{\mu\nu}e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k\sigma^{\mu\nu}u_t\right)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching $\mathscr{L}$ to UV-completed NP models
- all new physics models match a finite number of operators at each order in $1/\Lambda$

Weak-scale Effective Theories: flavor conserving

★ Effective Lagrangians parameterize New Physics without specifying a model

- write out all terms consistent with symmetries and according to power counting
- match to SMEFT/possible UV completions and compute experimental observables

$$\begin{aligned}\mathcal{L}_{\ell_1\to\ell_2\nu_2\bar\nu_1} = -\frac{4G_F}{\sqrt{2}}\Big[& g^S_{RR}\left(\overline{\ell_2}_R\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_L\ell_{1R}\right) + g^S_{RL}\left(\overline{\ell_2}_R\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\ell_{1L}\right)\\ &+ g^S_{LR}\left(\overline{\ell_2}_L\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\ell_{1R}\right) + g^S_{LL}\left(\overline{\ell_2}_L\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_R\ell_{1L}\right)\\ &+ g^V_{RR}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_R\gamma_\alpha\ell_{1R}\right) + g^V_{RL}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\gamma_\alpha\ell_{1L}\right)\\ &+ g^V_{LR}\left(\overline{\ell_2}_L\gamma^\alpha\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\gamma_\alpha\ell_{1R}\right) + g^V_{LL}\left(\overline{\ell_2}_R\gamma^\alpha\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\gamma_\alpha\ell_{1L}\right)\\ &+\frac{g^T_{RL}}{2}\left(\overline{\ell_2}_R\sigma_{\alpha\beta}\nu_{\ell_2 L}\right)\left(\overline{\nu_{\ell_1}}_R\sigma^{\alpha\beta}\ell_{1L}\right) + \frac{g^T_{LR}}{2}\left(\overline{\ell_2}_L\sigma_{\alpha\beta}\nu_{\ell_2 R}\right)\left(\overline{\nu_{\ell_1}}_L\sigma^{\alpha\beta}\ell_{1R}\right) + h.c.\Big],\end{aligned}$$

- which for $\mu \to e\nu\bar\nu$ (muon decay) leads to

$$\begin{aligned}\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}&\left[F\left(\frac{m_e^2}{m_\mu^2}\right) + 4\eta\frac{m_e}{m_\mu}G\left(\frac{m_e^2}{m_\mu^2}\right) - \frac{32}{3}\frac{m_e^2}{m_\mu^2}\left(\rho-\frac{3}{4}\right)\left(1-\frac{m_e^4}{m_\mu^4}\right)\right]\\ &\times\left(1+\frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)\left[1+\frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right],\end{aligned}$$

- where ρ and η are the Michel parameters

$$\rho = \frac{3}{16}\left[\left|g^S_{RR}\right|^2 + \left|g^S_{LL}\right|^2 + \left|g^S_{RL}-2g^T_{RL}\right|^2 + \left|g^S_{LR}-2g^T_{LR}\right|^2 + \frac{3}{4}\left(\left|g^V_{RR}\right|^2+\left|g^V_{LL}\right|^2\right)\right],$$

$$\eta = \frac{1}{2}\mathrm{Re}\left[g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR} + g^V_{RL}\left(g^{S*}_{LR}+6g^{T*}_{LR}\right) + g^V_{LR}\left(g^{S*}_{RL}+6g^{T*}_{RL}\right)\right],$$

Weak-scale Effective Theories: flavor violating

- Effective Lagrangians for $\Delta L_\mu = 0$, $\Delta L_\mu = 1$, and $\Delta L_\mu = 2$
  - SM: $$\mathcal{L}_{\rm eff}^{\Delta L_\mu = 0} = -\frac{4G_F}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_e}_L \gamma^\alpha \nu_{\mu L}\right)$$
  - four-fermion operators (assume no FCNC in quark currents for now)

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu = 1} = -\frac{1}{\Lambda^2} \sum_f \Big[\left(C_{VR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{VL}^f \, \overline{\mu}_L \gamma^\alpha e_L\right) \, \overline{f} \gamma_\alpha f$$
$$+ \left(C_{AR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{AL}^q \, \overline{\mu}_L \gamma^\alpha e_L\right) \, \overline{f} \gamma_\alpha \gamma_5 f$$
$$+ m_e m_f G_F \left(C_{SR}^f \, \overline{\mu}_R e_L + C_{SL}^f \, \overline{\mu}_L e_R\right) \, \overline{f} f$$
$$+ m_e m_f G_F \left(C_{PR}^f \, \overline{\mu}_R e_L + C_{PL}^f \, \overline{\mu}_L e_R\right) \, \overline{f} \gamma_5 f$$
$$+ m_e m_f G_F \left(C_{TR}^f \, \overline{\mu}_R \sigma^{\alpha\beta} e_L + C_{TL}^f \, \overline{\mu}_L \sigma^{\alpha\beta} e_R\right) \, \overline{f} \sigma_{\alpha\beta} f + h.c. \Big],$$

  - dipole operators $$\mathcal{L}_D = -\frac{m_2}{\Lambda^2} \left[\left(C_{DR} \overline{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{DR} \overline{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) F_{\mu\nu} + h.c. \right]$$
  - gluonic (Rayleigh) operators $$\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2} \frac{\beta_L}{4\alpha_s} \Big[\Big(C_{GR} \overline{\ell}_1 P_R \ell_2 + C_{GL} \overline{\ell}_1 P_L \ell_2 \Big) G_{\mu\nu}^a G^{a\mu\nu}$$
$$+ \Big(C_{\tilde{G}R} \overline{\ell}_1 P_R \ell_2 + C_{\tilde{G}L} \overline{\ell}_1 P_L \ell_2 \Big) G_{\mu\nu}^a \widetilde{G}^{a\mu\nu} + h.c. \Big]$$

AAP and D. Zhuridov

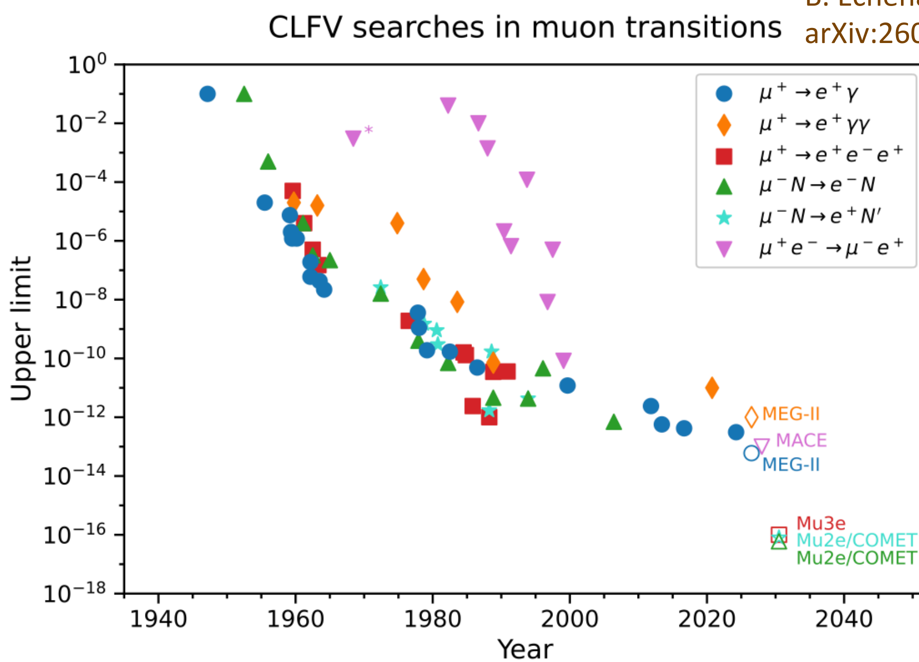
Lepton flavor violation: experiment

★ Leptons can help solve the most fundamental problems in particle physics! Flavor?

★ Possible experimental searches for Charged Lepton Flavor Violation (CLFV)

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+ e^- \to e^- \mu^+$ (muonium oscillations)
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm2) + e^\mp e^\mp$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

B. Echenard, AAP arXiv:2602.12442



★ Decays are highly suppressed in the Standard Model: $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu_i}}{M^2_W}\right|^2 < 10^{-54}$

★ But: no trivial FCNC vertices in the Standard Model: sensitive tests of New Physics!

Flavor violation and effective Lagrangians

★ Radiative FCNC decays of leptons $\ell_1 \to \ell_2 + \gamma$

- the most general amplitude is

$$A_{\ell_1 \to \ell_2 \gamma}(p, p') = \frac{i}{m_{\ell_1}} \overline{u}_{\ell_2}(p') \left[A_L P_L + A_R P_R\right] \sigma_{\mu\nu} q^\nu u_{\ell_1}(p) \epsilon^{*\mu},$$

- which leads to the decay rate

$$\Gamma(\ell_1 \to \ell_2 \gamma) = \frac{m_{\ell_1}}{16\pi} \left(|A_L|^2 + |A_R|^2 \right)$$

with $A_R = A_L^* = \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} \left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi} \right) \equiv \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} C_\gamma^{fi}$

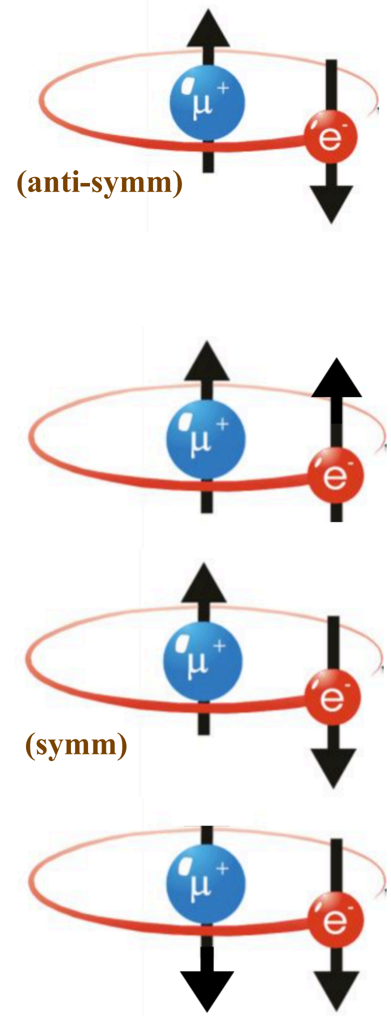
Effective coupling (example)	Bounds on Λ (TeV) (for $\lvert\mathcal{C}_{ij}^6\rvert = 1$)	Bounds on $\lvert\mathcal{C}_{ij}^6\rvert$ (for $\Lambda = 1$ TeV)	Observable
$\mathcal{C}_{e\gamma}^{\mu e}$	6.3×10^4	2.5×10^{-10}	$\mu \to e\gamma$
$\mathcal{C}_{e\gamma}^{\tau e}$	6.5×10^2	2.4×10^{-6}	$\tau \to e\gamma$
$\mathcal{C}_{e\gamma}^{\tau\mu}$	6.1×10^2	2.7×10^{-6}	$\tau \to \mu\gamma$
$\mathcal{C}_{\ell\ell,ee}^{\mu eee}$	207	2.3×10^{-5}	$\mu \to 3e$
$\mathcal{C}_{\ell\ell,ee}^{e\tau ee}$	10.4	9.2×10^{-5}	$\tau \to 3e$
$\mathcal{C}_{\ell\ell,ee}^{\mu\tau\mu\mu}$	11.3	7.8×10^{-5}	$\tau \to 3\mu$
$\mathcal{C}_{(1,3)H\ell}^{\mu e}$, $\mathcal{C}_{He}^{\mu e}$	160	4×10^{-5}	$\mu \to 3e$
$\mathcal{C}_{(1,3)H\ell}^{\tau e}$, $\mathcal{C}_{He}^{\tau e}$	≈ 8	1.5×10^{-2}	$\tau \to 3e$
$\mathcal{C}_{(1,3)H\ell}^{\tau\mu}$, $\mathcal{C}_{He}^{\tau\mu}$	≈ 9	$\approx 10^{-2}$	$\tau \to 3\mu$

Teixeira; Feruglio, Paradisi, Pattori

Other interesting modes that probe similar couplings: $\ell_1 \to \ell_2\gamma\gamma$, $\ell_1 \to 3\ell_2$, and others

ORNL advantage: bound states: muonium

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is *true muonium*
- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu\nu_e$
- Muonium's been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches: decays, oscillations



Spin-0 (singlet) paramuonium

Spin-1 (triplet) orthomuonium

Hughes (1960)

The masses of singlet and triplet are almost the same!

Why study decays of muonium?

- Muon decay $\mu \to 3e$:

$$\begin{aligned}
\Gamma\ & (\mu \to 3e) = \\
= & \frac{\alpha m_\mu^5}{3\Lambda^4(4\pi)^2}\left(|C_{DL}|^2+|C_{DR}|^2\right)\left(8\log\left[\frac{m_\mu}{m_e}\right]-11\right) \\
+ & \frac{4m_\mu^5}{3\Lambda^4(16\pi)^3}\left(m_e^4 G_F^2\left(|C_{SR}^e|^2+|C_{SL}^e|^2\right)\right. \\
+ & 2\left(2\left(|C_{VR}^e|^2+|C_{VL}^e|^2+|C_{AR}^e|^2+|C_{AL}^e|^2\right)+|C_{AR}^e+C_{VR}^e|^2+|C_{AL}^e-C_{VL}^e|^2\right)) \\
- & \frac{\sqrt{4\pi\alpha}m_\mu^5}{3\Lambda^4(4\pi)^3}\left(\Re\left[C_{DL}\left(3C_{VR}^e+C_{AR}^e\right)^*\right]+\Re\left[C_R^D\left(3C_{VL}^e-C_{AL}^e\right)^*\right]\right)
\end{aligned}$$

- Muonium decay $M_\mu^V \to e^+e^-$:

$$\begin{aligned}
\Gamma\left(M_\mu^V \to e^+e^-\right) = & \frac{f_M^2 M_M^3}{48\pi\Lambda^4}\left\{\frac{3}{2}|C_{VR}^e+C_{AR}^e|^2-\frac{3}{2}|C_{VL}^e+C_{AL}^e|^2\right. \\
& \left.+|2C_{VL}^e+C_{VR}^e|^2+|2C_{AL}^e+C_{AR}^e|^2\right\}
\end{aligned}$$

- Note: different combination of Wilson coefficients!

R. Conlin, J. Osborne, AAP

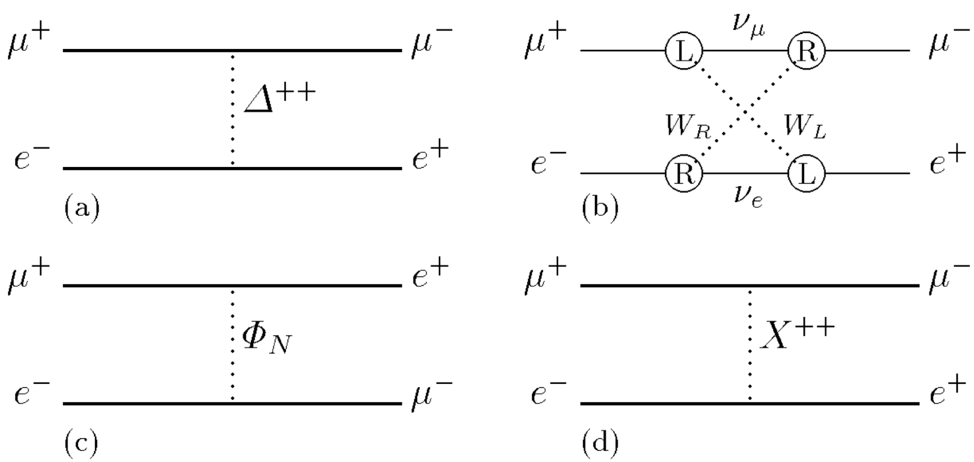
Muonium oscillations: just like $K^0\bar{K}^0$ mixing, but simpler!

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - … but there are plenty of New Physics models where it can happen

Clark, Love; Cvetic et al,
Li, Schmidt; Endo, Iguro, Kitahara;
Fukuyama, Mimura, Uesaka; …



$$\sim \left(\bar{\mu}\Gamma e\right)\left(\bar{\mu}\Gamma e\right)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but look for the decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

- If there is an interaction that couples M_μ and $\overline{M}_\mu$ (both SM or NP)
  - combined time evolution: non-diagonal Hamiltonian!

$$i\frac{d}{dt}\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix}$$

  - diagonalization: new mass eigenstates:

$$|M_{\mu_{1,2}}\rangle = \frac{1}{\sqrt{2}}\left[|M_\mu\rangle \mp |\overline{M}_\mu\rangle\right]$$

  - new mass eigenstates: mass and lifetime differences

$$\left.\begin{array}{l} \Delta m \equiv M_1 - M_2, \\ \Delta\Gamma \equiv \Gamma_2 - \Gamma_1. \end{array}\right\} \quad x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

(small)

These mass and width difference are observable quantities

Combined evolution = flavor oscillations

- Study oscillations via decays: amplitudes for $M_\mu \to f$ and $\overline{M}_\mu \to \bar{f}$
  - possibility of flavor oscillations ($M_\mu \to \overline{M}_\mu \to \bar{f}$)

$$\begin{aligned} |M(t)\rangle &= g_+(t)\,|M_\mu\rangle + g_-(t)\,|\overline{M}_\mu\rangle\,, \\ |\overline{M}(t)\rangle &= g_-(t)\,|M_\mu\rangle + g_+(t)\,|\overline{M}_\mu\rangle\,, \end{aligned}$$

with

$$\begin{aligned} g_+(t) &= e^{-\Gamma_1 t/2} e^{-i m_1 t}\left[1 + \frac{1}{8}(y - ix)^2\,(\Gamma t)^2\right], \\ g_-(t) &= \frac{1}{2} e^{-\Gamma_1 t/2} e^{-i m_1 t}\,(y - ix)\,(\Gamma t)\,. \end{aligned}$$

  - time-dependent width: $\Gamma(M_\mu \to \bar{f})(t) = \frac{1}{2} N_f \left|A_f\right|^2 e^{-\Gamma t} (\Gamma t)^2 R_M(x,y)$
  - oscillation probability: $P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \bar{f})}{\Gamma(M_\mu \to f)} = R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)$

R. Conlin and AAP

Oscillation parameters: introduction

- Mixing parameters are related to off-diagonal matrix elements
  - heavy and light intermediate degrees of freedom

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M}_\mu \left|\mathcal{H}_{\text{eff}}\right| M_\mu \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M}_\mu \left|\mathcal{H}_{\text{eff}}\right| n \right\rangle \left\langle n \left|\mathcal{H}_{\text{eff}}\right| M_\mu \right\rangle}{M_M - E_n + i\epsilon}$$

Local at scale $\mu = M_\mu$: only Δm
lepton number change $\Delta L_\mu = 2$

Bi-local at scale $\mu = M_\mu$: both Δm and $\Delta\Gamma$
lepton number changes: $(\Delta L_\mu = 1)^2$
or $(\Delta L_\mu = 0)(\Delta L_\mu = 2)$

  - each term has contributions from different effective Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{\Delta L_\mu = 0} + \mathcal{L}_{\text{eff}}^{\Delta L_\mu = 1} + \mathcal{L}_{\text{eff}}^{\Delta L_\mu = 2}$$

  - … all of which have a form $\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda^2}\sum_i c_i(\mu) Q_i,$ with $\Lambda \sim \mathcal{O}(TeV)$

Mass difference = real (dispersive) part; width difference: imaginary (absorptive) part

Oscillation parameters: mass difference

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\text{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\text{eff}}\right|M_\mu\rangle + \langle\overline{M}_\mu\left|i\int d^4x\ \text{T}\left[\mathcal{H}_{\text{eff}}(x)\mathcal{H}_{\text{eff}}(0)\right]\right|M_\mu\rangle\right]$$

  - consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

  - leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned} Q_1 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_L\gamma^\alpha e_L), \quad Q_2 = (\overline{\mu}_R\gamma_\alpha e_R)(\overline{\mu}_R\gamma^\alpha e_R), \\ Q_3 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_R\gamma^\alpha e_R), \quad Q_4 = (\overline{\mu}_L e_R)(\overline{\mu}_L e_R), \\ Q_5 &= (\overline{\mu}_R e_L)(\overline{\mu}_R e_L). \end{aligned}$$

  - need to compute matrix elements for both singlet and triplet states

Mass difference: results

- Spin-singlet muonium state:
  - matrix elements:

$$\langle \bar{M}_\mu^P | Q_1 | M_\mu^P \rangle = f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_2 | M_\mu^P \rangle = f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_3 | M_\mu^P \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_4 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_5 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2.$$

$$x_P = \frac{4(m_{red}\alpha)^3}{\pi\Lambda^2\Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} - \frac{3}{2}C_3^{\Delta L=2} - \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]$$

- Spin-triplet muonium state:
  - matrix elements

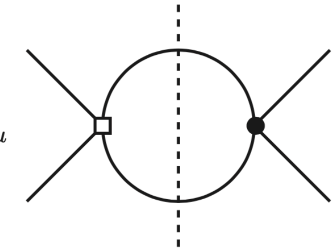
$$\langle \bar{M}_\mu^V | Q_1 | M_\mu^V \rangle = -3 f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_2 | M_\mu^V \rangle = -3 f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_3 | M_\mu^V \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_4 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_5 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2.$$

$$x_V = -\frac{12(m_{red}\alpha)^3}{\pi\Lambda^2\Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} + \frac{1}{2}C_3^{\Delta L=2} + \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]$$

Experimental constraints on x result in experimental constraints on Wilson coefficients $C_k^{\Delta L=2}$ that encode all information about possible New Physics contributions

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Width difference and muonium decays



- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution

$$\Gamma(M_\mu^V \to \bar{\nu}_\mu \nu_e) = \frac{G_F^2 f_M^2 M_M^3}{12\pi} \qquad Br(M_\mu^V \to \bar{\nu}_\mu \nu_e) = 8.8 \times 10^{-12}$$

AAP, R. Conlin, C. Grant

- Muonium two- and three-body decays
  - two-body decays ($M_\mu^{V,P} \to e^+e^-, \gamma\gamma, etc$) are dominated by New Physics
  - probe different combinations of SM EFT Wilson coefficients
    - e.g. $\mu \to 3e$ vs. $M_\mu \to e^+e^-$ (also phase space enhancement)

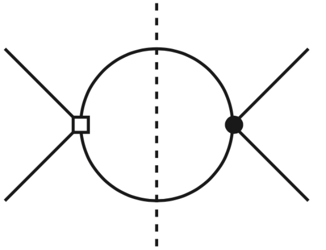
R. Conlin, J. Osborne, AAP

  - can $M_\mu \to invisible$ (SM: $M_\mu \to \nu_e \bar{\nu}_\mu$) be measured?

Gninenko, Krasnikov, Matveeev.
Phys.Rev. D87 (2013) 015016

Oscillation parameters: width difference

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



$$y = \frac{1}{2M_M\Gamma}\text{Im}\left[\langle\overline{M}_\mu\left| i\int d^4x\ \text{T}\left[\mathcal{H}_{\text{eff}}(x)\mathcal{H}_{\text{eff}}(0)\right]\right| M_\mu\rangle\right]$$

$$= \frac{1}{M_M\Gamma}\text{Im}\left[\langle\overline{M}_\mu\left| i\int d^4x\ \text{T}\left[\mathcal{H}_{\text{eff}}^{\Delta L_\mu=2}(x)\mathcal{H}_{\text{eff}}^{\Delta L_\mu=0}(0)\right]\right| M_\mu\rangle\right]$$

New Physics $\Delta L_\mu = 2$ contribution

Standard Model $\Delta L_\mu = 0$ contribution

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

$$Q_6 = \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right),$$

$$Q_7 = \left(\overline{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right)$$

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}}\left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_e}_L\gamma^\alpha\nu_{\mu L}\right)$$

$$\Gamma(M_\mu^V \to \overline{\nu}_e\nu_\mu) = \frac{f_M^2 M_M^3}{9\pi\Lambda^4}\left|C_6^{\Delta L_\mu=2} + C_7^{\Delta L_\mu=2}\right|^2 \quad \Bigg\| \quad \Gamma(M_\mu^V \to \overline{\nu}_\mu\nu_e) = \frac{G_F^2 f_M^2 M_M^3}{12\pi}$$

Width difference: results

- Spin-singlet muonium state:

$$y_P = \frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(C_6^{\Delta L=2} - C_7^{\Delta L=2}\right)$$

- Spin-triplet muonium state:

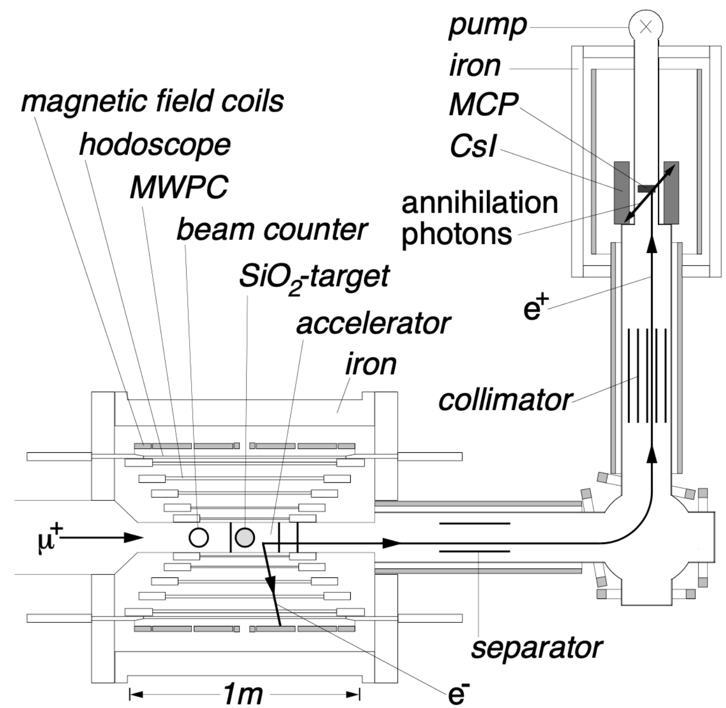
$$y_V = -\frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(5C_6^{\Delta L=2} + C_7^{\Delta L=2}\right)$$

- Note: y has the same $1/\Lambda^2$ suppression as the mass difference!

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Oscillations: experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - … which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

- MACS: observed 5.7×10^{10} muonium atoms after 4 months of running
  - magnetic field is taken into account (suppression factor)

Interaction type	2.8 μT	0.1 T	100 T
SS	0.75	0.50	0.50
PP	1.0	0.9	0.50
$(V \pm A) \times (V \pm A)$ or $(S \pm P) \times (S \pm P)$	0.75	0.35	0.0
$(V \pm A) \times (V \mp A)$ or $(S \pm P) \times (S \mp P)$	0.95	0.78	0.67

L. Willmann, et al. PRL 82 (1999) 49

  - no oscillations have been observed (yet!)

Experimental constraints

- We can now put constraints on the Wilson coefficients of effective operators from experimental data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P(M_\mu{}^i \to \overline{M}_\mu{}^i)$$

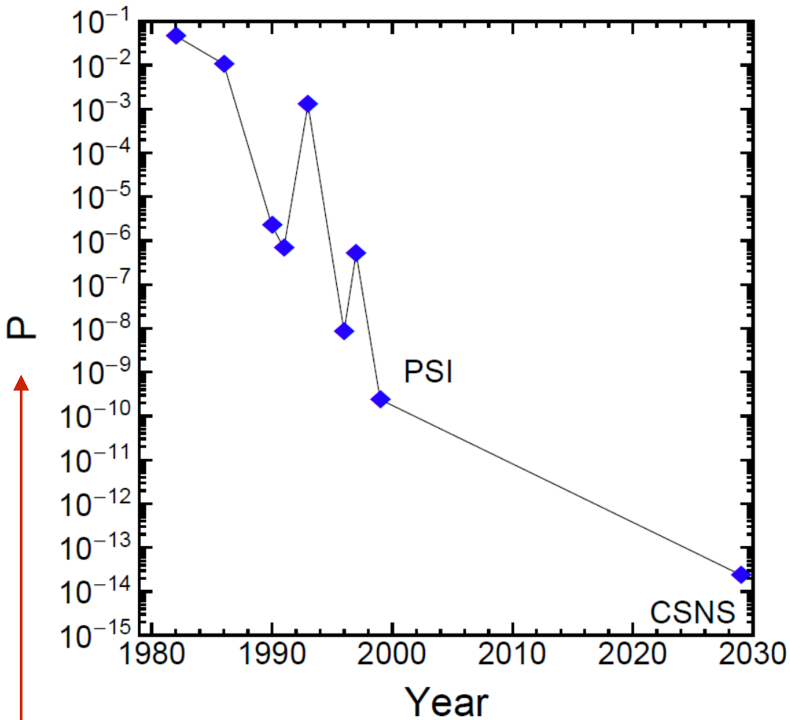
  - set Wilson coefficients to one, set constraints on the scale probed

Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

Probing LHC scales in a table-top setup!

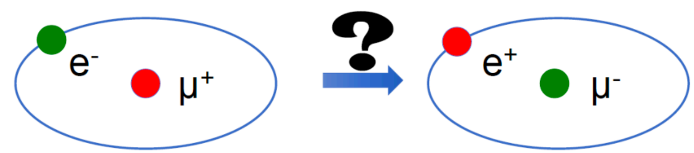
R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Fundamental science with EMuS (China)



$$P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y)$$

$$R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)$$



- The latest bound was done at PSI more than 20 years ago with a muon intensity $8 \times 10^6 \mu^+/s$ and high-precision magnetic spectrometer.
- Timing resolution in detector: ~ ns
- Position resolution in detector: ~ mm
- EMuS plan to offer $10^9 \mu^+/s$
- Current timing resolution in detector: ~ ps
- Current position resolution in detector:~μs
- Expect to be improved by > O(10^2)?

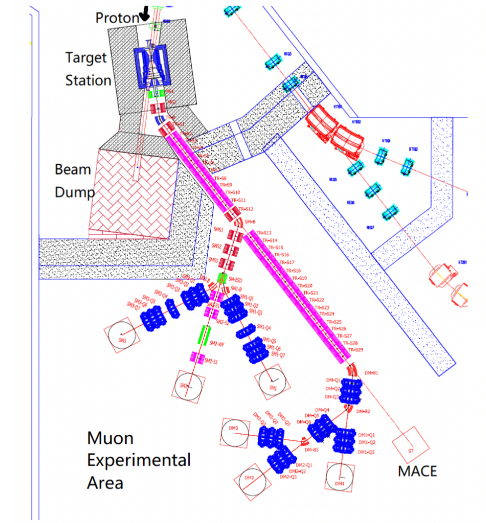
MACE experiment at EMuS (Chinese SNS)
Jian Tang, talk at RPPM meeting (Snowmass 2021)

Conceptual Design of the Muonium-to-Antimuonium Conversion Experiment (MACE)
A.-Y. Bai, …, AAP, …, Nucl. Sci. Tech. 37 (2026) 4, 57

New muon sources: CSNS

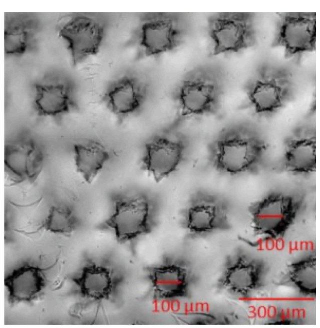
- Experimental Muon Source (EMuS) at Chinese Spallation Neutron Source
  - CSNS proton driver can be used to produce muons

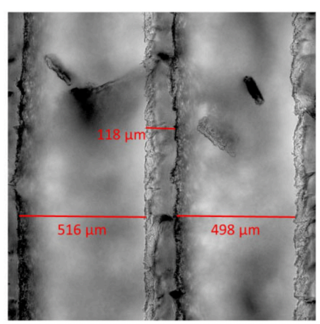
	Proton driver [MW]	Intensity [$\times 10^6$/s]	Polarization[%]	Spread [%]
PSI	1.30	420	90	10
ISIS	0.16	1.5	95	≤ 15
RIKEN/RAL	0.16	0.8	95	≤ 15
JPARC	1.00	100	95	15
TRIUMF	0.075	1.4	90	7
EMuS	0.025	83	50	10



  - EMuS will produce up to $10^9\ \mu^+/s$, which will be transported to MACE

- Muonium states will be formed in laser-ablated silica aerogel target





  - the muonium emission rate of the aerogel target with holes is up to 36 times higher than that of the silica powder target used in MACS

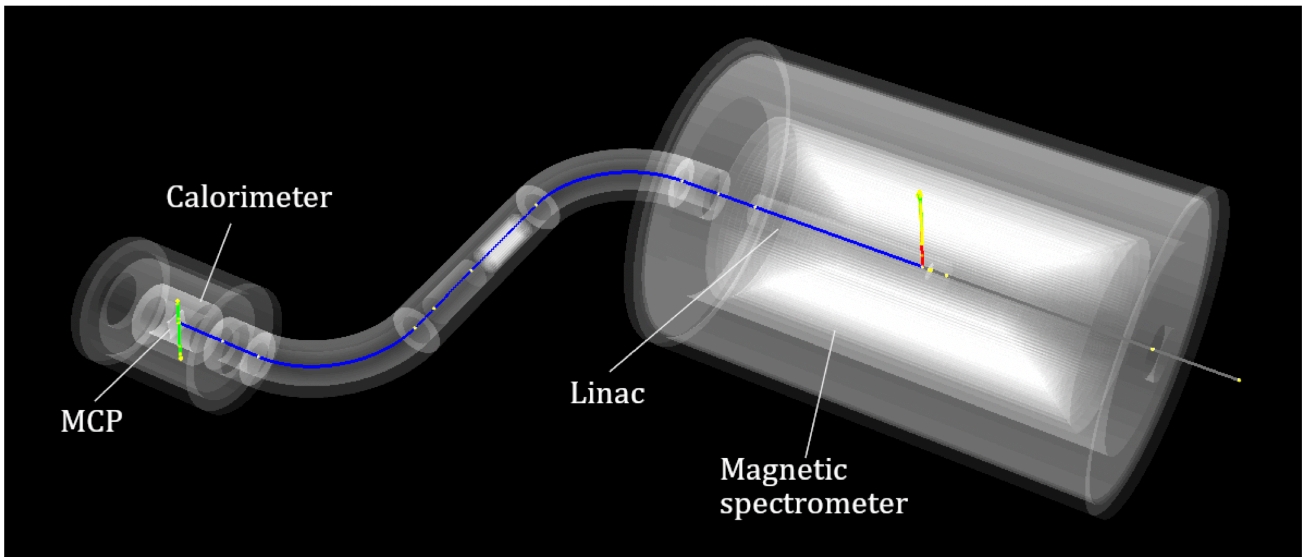
J. Beare et al, Prog. Theor. Exp. Phys. 2020, 123C01

Possible competition: MACE experiment

- Proposal: Muonium-to-Antimuonium Conversion Experiment (MACE)

A.-Y. Bai, …, AAP, …, 2410.18817 [hep-ex]

  - MACE uses the same kinematical tag as MACS
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast (Michel) e^- of 52.8 MeV and slow (shell) e^+ of 13.5 eV



  - Triple coincidence: The Michel electron is detected by the drift chamber. The atomic-shell positron is accelerated and transported to the MCP and annihilates into two photons. The photons are detected by the electromagnetic calorimeter.

Conclusions and things to take home

- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Muonium is the simplest atom: atomic physics
  - level splitting (Lamb shift): probe NP w/out QCD complications

MuSEUM experiment (J-PARC)

- Muons are ideal tools to probe fundamental physics
  - flavor-conserving quantities (g-2, EDM)
  - flavor-changing neutral current decays
  - flavor oscillations (muonium-antimuonium conversion)
  - muon transitions already probe the LHC energy domain and can do better!

Prospects for precise predictions of a_μ in the Standard Model
G. Colangelo, et. al., arXiv:2203.15810 [hep-ph]

- New experimental facilities are needed: ORNL?

Conceptual Design of the Muonium-to-Antimuonium Conversion Experiment (MACE)
A.-Y. Bai, …, AAP, …, 2410.18817 [hep-ex]


EXTRA!

Mass difference: matrix elements

- QED bound state: know leading order wave function!
  - spacial part is the same as in Hydrogen atom

$$\varphi(r) = \frac{1}{\sqrt{\pi a_{M_\mu}^3}} e^{-\frac{r}{a_{M_\mu}}}$$

  - can unambiguously compute decay constants and mixing MEs (QED)

$$\langle 0 | \overline{\mu}\gamma^\alpha\gamma^5 e \left| M_\mu^P \right\rangle = i f_P p^\alpha, \quad \langle 0 | \overline{\mu}\gamma^\alpha e \left| M_\mu^V \right\rangle = f_V M_M \epsilon^\alpha(p),$$

$$\langle 0 | \overline{\mu}\sigma^{\alpha\beta} e \left| M_\mu^V \right\rangle = i f_T \left(\epsilon^\alpha p^\beta - \epsilon^\beta p^\alpha\right),$$

  - in the non-relativistic limit all decay constants $f_P = f_V = f_T = f_M$

$$f_M^2 = 4\frac{|\varphi(0)|^2}{M_M}$$

(QED version of Van Royen-Weisskopf)

  - NR matrix elements: “vacuum insertion” = direct computation

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell} \overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2 M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\text{eff}}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$

Chang, Keung (89);
Schwartz (89);
Han, Tang, Zhang (21)

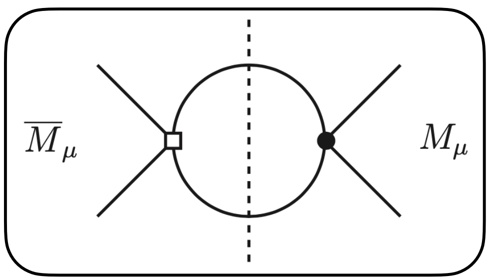
Is it better than/worse than/complimentary to $\mu \to 3e$?

Effective Lagrangians and lifetime difference

- Effective Lagrangians for $\Delta L_\mu = 0$, $\Delta L_\mu = 1$, and $\Delta L_\mu = 2$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_e}_L \gamma^\alpha \nu_{\mu L}\right)$$

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = & -\frac{1}{\Lambda^2}\sum_f \Big[\left(C_{VR}^f\, \overline{\mu}_R \gamma^\alpha e_R + C_{VL}^f\, \overline{\mu}_L \gamma^\alpha e_L\right)\ \overline{f}\gamma_\alpha f \\
& + \left(C_{AR}^f\, \overline{\mu}_R \gamma^\alpha e_R + C_{AL}^q\, \overline{\mu}_L \gamma^\alpha e_L\right)\ \overline{f}\gamma_\alpha \gamma_5 f \\
& + m_e m_f G_F \left(C_{SR}^f\, \overline{\mu}_R e_L + C_{SL}^f\, \overline{\mu}_L e_R\right)\ \overline{f} f \\
& + m_e m_f G_F \left(C_{PR}^f\, \overline{\mu}_R e_L + C_{PL}^f\, \overline{\mu}_L e_R\right)\ \overline{f}\gamma_5 f \\
& + m_e m_f G_F \left(C_{TR}^f\, \overline{\mu}_R \sigma^{\alpha\beta} e_L + C_{TL}^f\, \overline{\mu}_L \sigma^{\alpha\beta} e_R\right)\ \overline{f}\sigma_{\alpha\beta} f + h.c. \Big],
\end{aligned}$$



$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu) Q_i(\mu)$$

$$Q_6 = \left(\overline{\mu}_L \gamma_\alpha e_L\right)\left(\overline{\nu_\mu}_L \gamma^\alpha \nu_{eL}\right), \quad Q_7 = \left(\overline{\mu}_R \gamma_\alpha e_R\right)\left(\overline{\nu_\mu}_L \gamma^\alpha \nu_{eL}\right)$$

- $\Delta\Gamma$: naively $\mathcal{O}(\Lambda^{-4})$ from double $\Delta L_\mu = 1$ insertion! But not always…

- A possibility of using muon beams at CMP facilities

Jian Tang, talk at RPPM meeting (Snowmass 2021)

	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

×5 CSNS-II upgrade

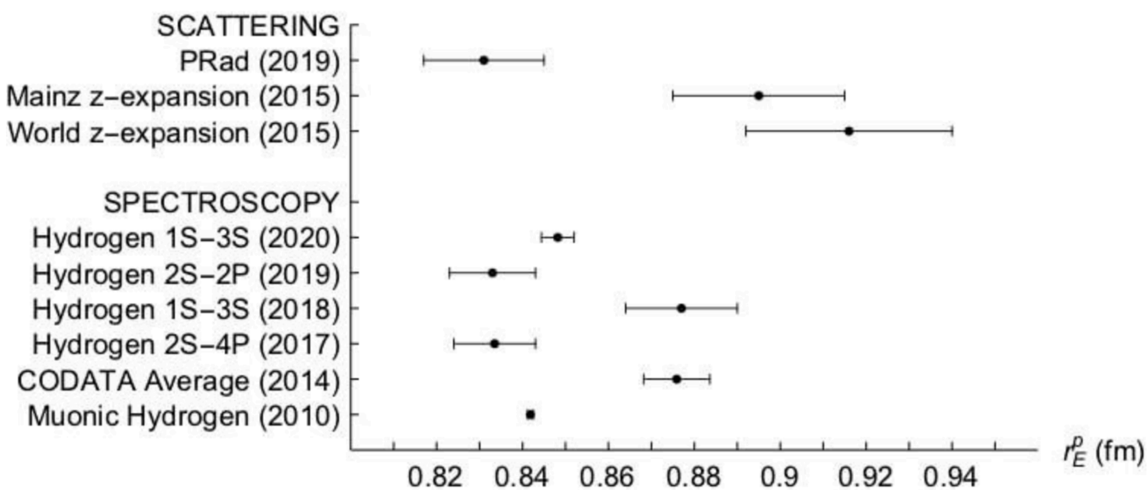
Facility	Source Type	Intensity (μ+/sec)*
ISIS	pulsed	1.5×10^6
J-PARC	continuous	1.8×10^6
PSI	continuous	7.0×10^4
TRIUMF	pulsed	5.0×10^6
SEEMS	pulsed	1.9×10^8

- Muonium Antimuonium Conversion Experiment (MACE) EMuS at CSNS

Muons and recent experimental anomalies

★ Proton's radius from muonic hydrogen: possible New Physics?

★ Level splittings (e.g. Lamb shift) are sensitive to the charge radius of the proton



★ They are also sensitive to QED radiative corrections

★ Are there possible light New Physics particles that are responsible for this difference?

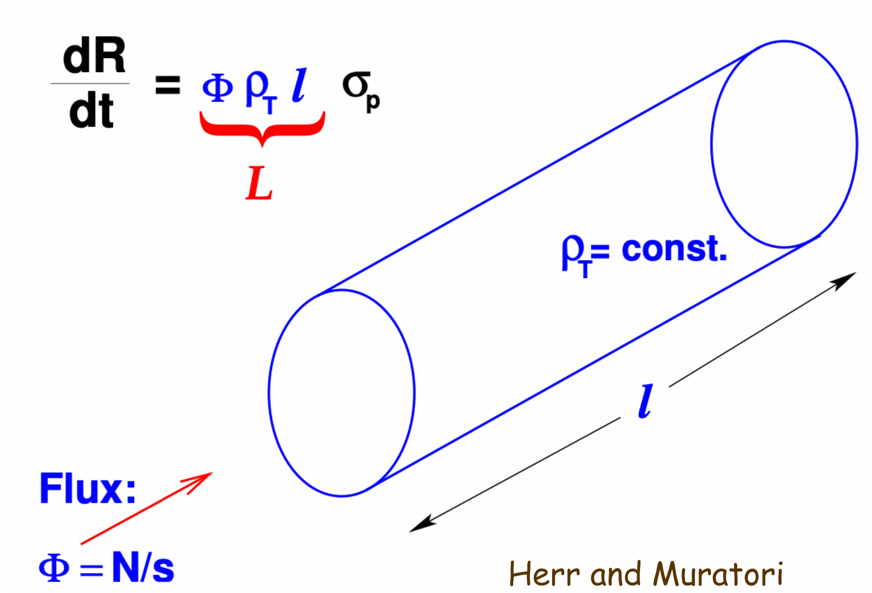
Barger et al, PRL 106 (2011) 153001

Remove proton radius issue from the problem: atomic physics with muonium?

Experimental studies of rare processes: luminosity

★ Need a lot of muons: high luminosity experiments

– Number of events/second



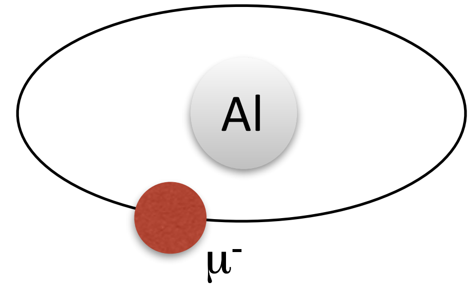
	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$
Tevatron ($p\bar{p}$)	1000x1000	$50\ 10^{30}$
HERA (e^+p)	30x920	$40\ 10^{30}$
LHC (pp)	7000x7000	$10000\ 10^{30}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$
PEP (e^+e^-)	9x3	$3000\ 10^{30}$
KEKB (e^+e^-)	8x3.5	$10000\ 10^{30}$
eRHIC		10^{33}-10^{35}

– ... or another way $$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$

What if incident particles formed bound states with target particles?

Bound states: muon conversion

★ Basic idea for the muon conversion experiment



★ take low energy muons (~ 30 MeV) and stop them in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

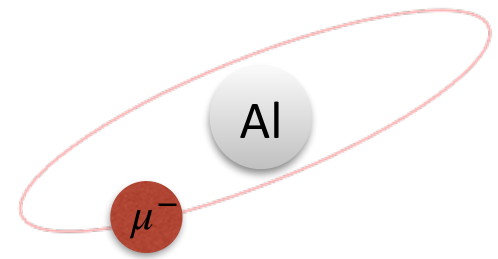
★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$

overlap probability: $p \sim r^{2\ell} Z^3$ ← large overlap for an s-wave and high-Z nucleus

Measure $R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A, Z-1)\right]}$ to probe NP

Bound states: muon conversion

- How effective is this approach compared to scattering?
  - let's compute effective luminosity
  - recall that



$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$

  - in this "experiment" the probability density is given by the 1s wave function
  - … and we need to take into account the fact that muon decays
  - Then luminosity = (density)(velocity)(flux of muons)(lifetime)

$$L_{\text{eff}} = |\psi(0)|^2 \times \alpha Z \times \Phi_\mu \times \tau_\mu = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \Phi_\mu \tau_\mu$$

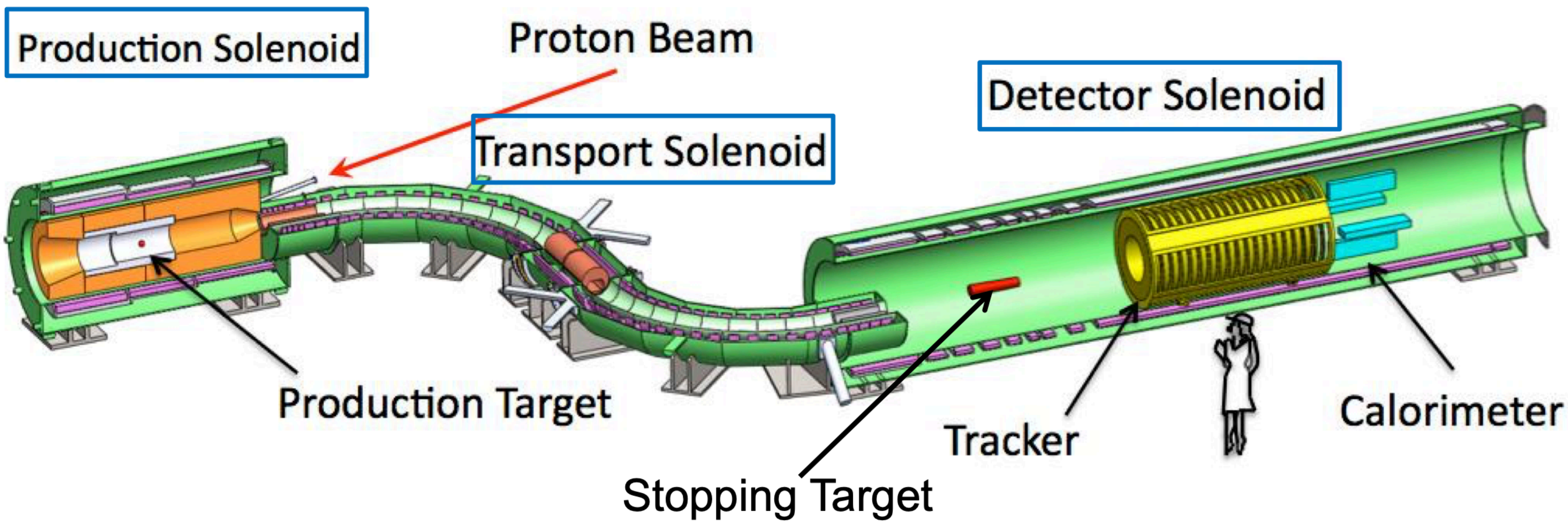
  - For Al target (Z=13), flux of $\Phi_\mu = 10^{10}$ muons/sec and $\tau_\mu = 2\ \mu$sec

$$L_{\text{eff}} = 10^{48} \text{cm}^{-2}\ \text{sec}^{-1}$$

Bernstein, Czarnecki

Muon conversion: Mu2e experiment

R. Ehrlich



- A pulsed proton beam hits the production target to produce pions which decay into muons.
- The muons get transported via the transport solenoid to the detector solenoid where they hit the aluminum stopping target.
- If conversion electrons are produced in the stopping target, they will move through the tracker to the calorimeter.

Mu2e will see a signal if as few as one in 100 quadrillion muons transforms into an electron!

Bound states: muon conversion

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

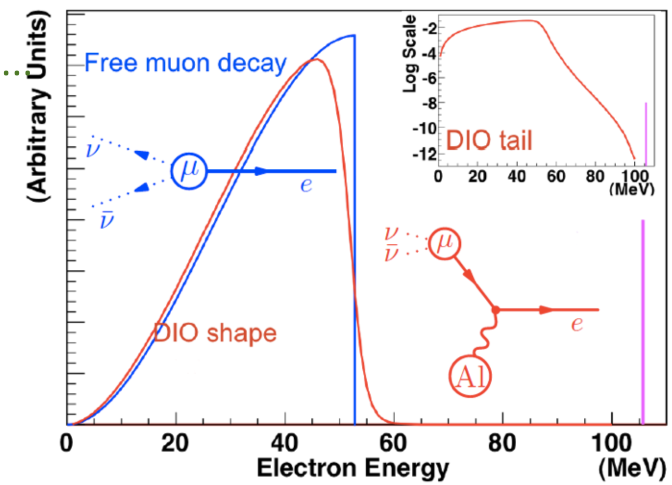
★ The experiment is tricky

- ✓ Muon conversion gives monoenergetic electrons...
- ✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

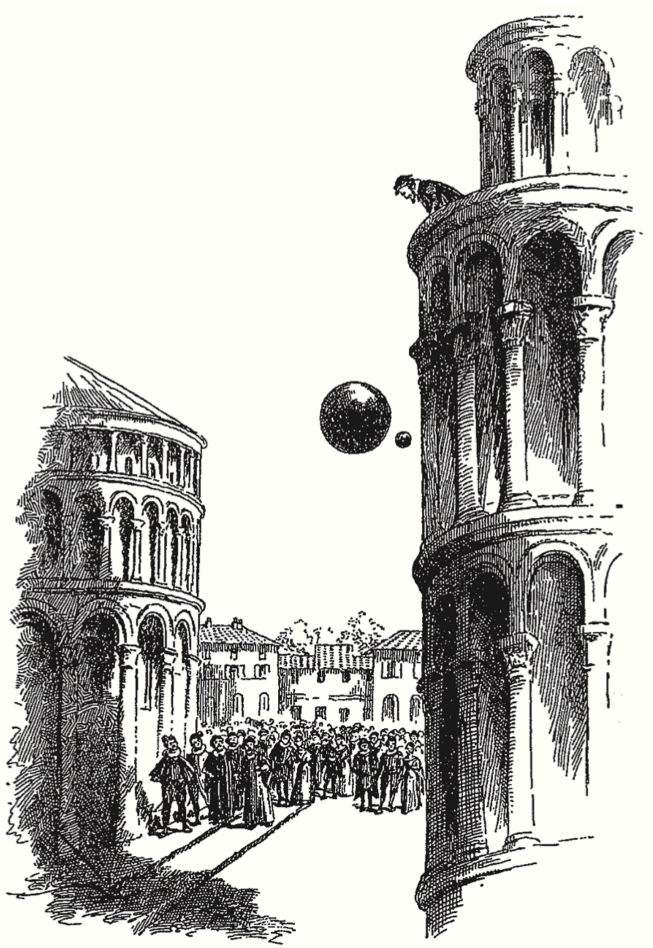
SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} <$ a few $\times 10^{-17}$



Czarnecki, Marciano, Tormo

★ Effective Lagrangians: let's do a familiar example: Galleo and gravity


'They were seen to fall evenly.'

– the ball is described by a Lagrange function

$$L = \frac{mv^2}{2} - V(h) = \frac{mv^2}{2} - mgh \quad \text{with } v = \partial h / \partial t$$

– only potential difference is physical: symmetry $h \to h + a$
equivalent statement: $F = mg$ does not depend on h

But this is not true!

– moving the tower on top of Mount Everest we change free-fall acceleration, as $g(R) = GM/R^2$, in fact,

$$R\frac{\partial}{\partial R} g(R) = \gamma_g g(R) \quad \text{with } \gamma_g = -2$$

– can we make the approximation better?

★ Newton's classical gravity as an effective theory

– let's assume that the tower height is always smaller than the Earth's radius, i.e. $h/R \to 0$

$$V(h) = C_1(R)m\left(\frac{h}{R}\right) + C_2(R)m\left(\frac{h}{R}\right)^2 + \ldots$$

– we introduced *power counting*: each term in (h/R) is smaller than the previous one, we can fit the observables to experiment to determine C_1 and C_2

Or we can do better: match to the full theory: $V(h) = G\dfrac{Mm}{r} = G\dfrac{Mm}{R+h}$

no manifest symmetry $h \to h + a$

– expand in $h/R \to 0$ and match
$$V(h) = G\frac{M}{R}m\left(\frac{h}{R}\right) - G\frac{M}{R}m\left(\frac{h}{R}\right)^2 + \ldots$$
$$= mgh - \frac{mg}{R}h^2 + \ldots$$

★ Effective theory approach: consistent order-by-order improvement

– relevant variables (h), power counting ($h/R \to 0$), matching to full theory or expimental data

Effective Lagrangians

★ How does this program work in Quantum Field Theory?

– introduce the notion of field dimension; in scalar QFT (natural units: $\hbar = c = 1$)

$$S = \int d^4x \, \mathcal{L} = \frac{1}{2} \int d^4x \left(\partial^\mu \phi \partial_\mu \phi - m^2 \phi^2 \right)$$

action is dimensionless

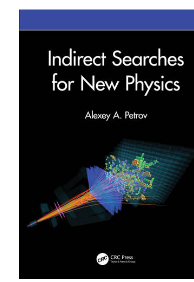
mass-dimension d=-4, as [x]=-1

must be mass dimension d=4

– since the mass dimension of a derivative is $\left[\partial\right] = +1$, we must conclude that $[\phi] = +1$

TABLE 1.1 Dimensions of fields and operators

Field or operator	Canonical dimension, $[E]$
Spin 0 field (ϕ, etc.)	1
Spin 1/2 field (ψ, etc.)	3/2
Spin 1 field (A_μ, etc.)	1
Derivative acting on a field (D_μ, etc.)	1
Mass term (m)	1
Lagrangian ($\mathcal{L}$)	4



– example: Fermi interaction (weak force)

$$\mathcal{L}_F = -\frac{4G_F}{\sqrt{2}} \left(\bar{\mu}_L \gamma_\alpha \nu_{\mu L} \right) \left(\bar{\nu}_{eL} \gamma_\alpha e_L \right) = -\frac{1}{2} \frac{g^2}{M_W^2} \left(\bar{\mu}_L \gamma_\alpha \nu_{\mu L} \right) \left(\bar{\nu}_{eL} \gamma_\alpha e_L \right)$$

$$d_{\mathcal{L}} = 4 \times \frac{3}{2} = 6$$